TEACHING EFFICACY AND KNOWLEDGE OF ALGEBRA FOR TEACHING AMONG SECONDARY MATHEMATICS TEACHERS

A Dissertation Presented to the
Faculty of the College of Education
University of Houston

In Partial Fulfillment
of the Requirements for the Degree

Doctor of Philosophy

by
Ashley Warren
December 2016

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A Dissertation for the Degree
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by
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Abstract

Research has revealed the importance of identifying specialized content knowledge essential for teaching mathematics (Ball, Thames, & Phelps, 2008), the significance of assessing mathematics teacher’s teaching efficacy and outcome expectancy beliefs (Enochs, Smith, & Huinker, 2000) and the necessity for all students to have access to algebra within an elementary and secondary mathematics curriculum (National Council for Teaching Mathematics, 2014b). Previous literature, however, has failed to address the relation of teaching efficacy, outcome expectancy, and teacher’s algebra content knowledge. The primary aim of the present study is to determine if scores on a knowledge of algebra for teaching assessment can be predicted by teachers’ levels of mathematics teaching efficacy and outcome expectancy. The secondary aim of the study is to investigate the factors accounting for secondary mathematics teacher’s teaching efficacy, outcome expectancy, and knowledge of algebra for teaching.

Data from an algebra content knowledge assessment, teaching efficacy and outcome expectancy beliefs survey, and demographic questionnaire were collected from a sample of 100 pre-service, in-service, and former secondary mathematics teachers. Quantitative data analysis methods including multiple regression, hierarchical regression, and mediation were employed to address the aims of the study. Additionally, principal components analysis was conducted to assess the psychometric properties of the mathematics teaching efficacy beliefs instrument (MTEBI) and knowledge of algebra for teaching (KAT) scales. The results indicated that each measure had satisfactory reliability.

Multiple regression analyses were used to determine if individual teacher characteristics were related to KAT and MTEBI scores. Results indicated that individual teacher characteristics (having a math teaching certification, increased years of education, an older age, increased years of teaching experience, being a high school teacher, a public school teacher, and ethnicity) were significant in predicting KAT and MTEBI scores. Hierarchical multiple regression analyses revealed that individual teacher characteristics (having a math teaching certification, increased years of education, an older age, increased years of teaching experience, and being a public school teacher) improved prediction of teaching efficacy and outcome expectancy beliefs while controlling KAT. Additionally, the mediating effects of KAT on teaching efficacy, outcome expectancy, and individual teacher characteristics were addressed using Hayes (2013)'s macro known as PROCESS. Results showed that being a public school teacher, exerted an indirect effect, via knowledge of algebra for teaching on outcome expectancy.

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Chapter I

Introduction

In recent years, substantial attention has been placed on recruiting, preparing, and supporting excellent Science, Technology, Engineering, and Mathematics (STEM) teachers in the United States. In President Barack Obama's 2011 State of the Union address, declarations of a 2015 Budget of $40 million to support the goal of preparing 100,000 excellent STEM teachers over the next decade (United States Department of Education, 2014) highlighted the importance of teacher education in these fields. Particularly, algebra has emerged as a filter that prevents many students from pursuing advanced mathematics courses in high school, which limits their preparation for college and/or selection of STEM careers (Brown, Davis, and Kulm, 2011).

According to the Department of Education (2010), only 16 percent of American high school seniors are proficient in mathematics and interested in STEM careers; additionally, the United States is falling behind internationally, ranking 25th in mathematics and 17th in science among industrialized nations. This has led to a persistent increase in the emphasis on STEM education among policy makers and school leaders in the United States. Mathematics achievement in particular within the secondary school setting has served as a baseline for identifying students' success. Unfortunately, as many students' complete high school and enter college they are required to enroll in remedial mathematics courses due to poor academic performance. Based on a report conducted by the National Center for Education Statistics in partnership with the United States Department of Education (2013), 19% of first-year college students in the 2007-2008 school year had taken at least one remedial mathematics course. Placement into remedial courses are a direct result of student's academic performance in secondary school, indicating that the student is not equipped with the necessary knowledge and/or skills to succeed in a standard college mathematics course. Additionally, mathematics knowledge is fundamental to success in the global economy as it is no longer the exclusive domain of scientists and engineers (Kim & Hodges, 2012). Therefore, these skills are essential for success in college, technical school, and opportunities for future job placement. Subsequently, interest in mathematics education has become a primary focus of many post-secondary institutions.

According to the National Council of Teachers of Mathematics (NCTM, 2014a) one of the guiding principles for an excellent mathematics program is that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential. The NCTM guiding principle emphasizing effective teaching and learning has caused states to develop stringent requirements in order to identify "highly qualified" secondary mathematics teachers. This additional pressure has placed great attention on secondary mathematics teachers as classroom teachers are seen as the main source for providing curriculum-based information to students. This reform perspective requires substantial paradigmatic shifts for many teachers, including changes in important constructs related to the teaching of mathematics such as beliefs, attitudes, and knowledge (Swars, Smith, Smith, & Hart, 2009).

To be considered highly qualified, middle and high school math teachers must demonstrate competency in a secondary mathematics. The criteria for demonstrating this competency may vary by state and teaching level (e.g. specific examination and having a degree in the subject matter they teach). Additionally, teachers must also demonstrate competency in pedagogy, typically as part of a teacher education program required for licensure. For secondary mathematics teachers, this may include classes related to content specific pedagogy as well as a practicum. Teachers who are highly qualified and certified to teach mathematics have stronger pedagogical and mathematics knowledge and are more likely to better understand how students best learn mathematics (Brown, Davis, and Kulm, 2011).

According to the United States Department of Education (2014), there are several states that demonstrate an immense need in mathematics. The state of Texas, for example, has exhibited a critical shortage of mathematics teachers from 1993-2015 with a dire need for mathematics teachers in middle and high school. Based on this information, one may deem that qualified mathematics teachers are difficult to acquire. Therefore, the mathematics teaching profession is faced with the ongoing challenge of addressing the critical shortage problem, as well as preparing a substantial amount of highly qualified teachers to teach mathematics in hopes of increasing overall student achievement in secondary mathematics courses.

The current state of mathematics education in the United States indicates the importance of considering both the content knowledge of secondary math teachers as well as their ability beliefs related to teaching mathematics. Prior studies have shown that a teacher's ability beliefs associated with teaching (efficacy) and outcome beliefs associated with teaching (outcome expectancy) have been related to student's achievement (Tschannen-Moran & Woolfolk-Hoy, 2001; Angle & Moseley, 2009). Additionally, studies have found that a teacher's content knowledge specific for teaching mathematics was a significant predictor of student gains on an achievement test (Hill, Rowan, & Ball, 2005), and was found to correlate significantly with effective mathematics teaching practices, as measured by students' test scores on standardized tests (McCrory et al., 2012). However, prior studies fail to identify the connection between specific content knowledge for teaching, teacher's efficacy beliefs, and expectancy beliefs as a means of demonstrating secondary mathematics teaching competence.

Statement of the Problem

Well-prepared mathematics teachers are crucial for secondary schools to remain competitive in the discussion of mathematics achievement. Locally, the elevated need for qualified mathematics teachers across the United States (United States Department of Education, 2014) has been demonstrated in recent years with many school districts engaging in mass hiring of secondary mathematics teachers in order to fill the large amount of teaching vacancies found in middle school and high school mathematics classrooms. Characteristics that qualify secondary mathematics teachers to teach students, however, are not limited to solely having a bachelor's degree and completing a teacher preparation program. Even and Ball (2009) assert that preparing and maintaining a high-quality, professional teaching force that can teach mathematics effectively is a worldwide challenge in which all researchers can benefit from a worldwide conversation.

In mathematics, a teacher's efficacy is a significant predictor of mathematics instructional strategies, and highly efficacious teachers are more effective mathematics teachers than teachers with lower efficacy (Swars, 2005) with highly efficacious teachers exerting a positive effect on student learning (Enochs, Smith, & Huinker, 2000). Previous research has linked teaching efficacy to student achievement (Hoy & Spero, 2005), student motivation (Midgley, Feldlaufer, & Eccles, 1989) and teacher retention (Perrachione, Petersen, and Rosser, 2008). As teaching efficacy has been found to effect student outcomes and a teacher's commitment to teaching, identifying characteristics of highly efficacious teachers has generated curiosity among many educational leaders. As educators gain pedagogical knowledge, they become more capable and confident in helping students extend and formalize their understanding of mathematical concepts, which can contribute to students' development of positive attitudes toward mathematics and an increase in teacher's sense of efficacy (Ontario Ministry of Education, 2011). For centuries, the teaching and learning of mathematics has been categorized into specific content areas (calculus, geometry, trigonometry, etc.), yet recently the notion of mathematics knowledge for teaching (Hill, Schilling, & Ball, 2004; Ball, Hill, and Bass, 2005) has been proposed. Mathematics knowledge for teaching has been linked to students' achievement (Hill et al., 2005; Hill et al., 2008), additionally, Swars, Smith, Smith, and Hart (2009) found a positive correlation between teacher's mathematics teaching efficacy beliefs and mathematical knowledge for teaching (MKT) measures.

Furthermore, knowledge of algebra for teaching has been identified as necessary for secondary mathematics teachers. The knowledge related to teaching algebra is important for several reasons; algebra is offered in both middle and high school, in many states students cannot be awarded a high school diploma without successful completion of a yearlong algebra course, and nationwide student achievement in algebra has been a topic of concern in relation to the United States' global STEM achievement in comparison to other industrialized nations. Consequently, as more students are taking algebra, more teachers are needed to teach algebra. The need for hiring and retaining qualified mathematics teachers across the United States further adds to the significance of studying the specific mathematics knowledge a secondary mathematics teacher should know.

To date, many teaching efficacy studies report a teacher's efficacy in relation to student performance outcomes such as classroom based assessment scores and statewide standardized test results. Though teaching efficacy appears to have a strong relation to teacher effectiveness, Raudenbush and colleagues (1992) emphasize that feelings of positive efficacy do not guarantee effective teaching, since teachers with high levels of perceived efficacy may lack the requisite knowledge or skills to be effective; but low feelings of efficacy almost certainly work against effective teaching by decreasing teacher's capability to cope with the uncertainties of classrooms. With that contention, it is imperative that teaching efficacy is not studied in isolation, but rather investigated alongside the requisite knowledge and skills needed for teaching. According to Swars, Smith, Smith, Hart, (2009) although many studies have examined individual components of teacher's beliefs, attitudes, and knowledge, few have simultaneously examined the interrelatedness of these components. Specifically, present gaps in the literature exist among examining teaching efficacy beliefs, outcome expectancy beliefs, and knowledge of algebra for teaching concurrently.

Purpose of the Study

The intent of this study was to investigate the relationship between teaching efficacy, outcome expectancy, and knowledge of algebra for teaching through exploring variations in survey results amongst pre-service, in-service, and former secondary mathematics teachers. Specifically, the study was designed to determine if higher scores on a knowledge of algebra for teaching assessment can be predicted by teachers' levels of mathematics teaching efficacy and outcome expectancy.

As the literature review shows, much research has been conducted regarding a teacher's teaching efficacy and outcome expectancy beliefs. Little research has been conducted to determine whether these beliefs are related to knowledge of algebra for teaching, thus an existing relationship is unclear. Knowledge of algebra for teaching (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) is a construct in its infancy, unlike the established teaching efficacy and outcome expectancy constructs which have been researched frequently. Because of this, the literature within the knowledge of algebra for teaching construct has been sparse. The present study is significant as it can contribute to the knowledge of algebra for teaching literature with a potential of identifying additional factors which contribute to a teacher's knowledge of algebra for teaching. Additionally, results of the study may provide implications for policy makers, district human resources departments, and secondary mathematics teacher preparation programs in efforts to support and encourage the professional development of secondary mathematics educators.

Research Questions and Hypotheses

In this study, the researcher examined pre-service, in-service, and former secondary mathematics teacher's responses to the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), designed to measure teaching efficacy and outcome expectancy. Additionally, a knowledge of algebra for teaching (KAT) survey was used to measure participants teaching knowledge. Presently, the KAT survey is seldom used in research as it is a newly developed instrument, thus the hypotheses in relation to KAT are based on inferences from comparable teaching knowledge surveys. The following research questions and hypotheses were addressed for exploration: Are there differences among secondary math teacher's teaching characteristics such as grade level taught, gender, ethnicity, years of experience, and certification, in terms of knowledge of algebra for teaching, teaching efficacy, and outcome expectancy? Do teacher characteristics and knowledge of algebra for teaching significantly predict teaching efficacy and outcome expectancy beliefs? Furthermore, what is the relation between teacher characteristics and teaching efficacy? Does knowledge of algebra for teaching mediate the relation between teacher characteristics and teaching efficacy? Lastly, what is the relation between teacher characteristics and outcome expectancy? Does knowledge of algebra for teaching mediate the relation between teacher characteristics and outcome expectancy?

H_1: It is hypothesized that high school teachers will demonstrate higher levels of teaching efficacy (Raudenbush, Rowan, & Cheong, 1992) than middle school teachers. It is also hypothesized that high school teachers will demonstrate higher knowledge of algebra for teaching (KAT) scores than middle school teachers based on earlier studies (Depaepe, et al., 2015; Hill & Lubienski, 2007) which investigated the mathematical knowledge for teaching (MKT) scores of elementary and middle school teachers where a higher grade level indicated better performance on MKT measures. It is also hypothesized that years of experience will not predict KAT scores as years of experience failed to predict MKT scores (Hill & Lubienski, 2007). It is hypothesized that KAT will be correlated with math teaching certification as Hill et al. (2005), found teachers MKT to be correlated with teacher's certification status.

H_2: It is hypothesized that teacher characteristics and knowledge of algebra for teaching (KAT) will predict teaching efficacy and outcome expectancy beliefs of middle and high school mathematics teachers. Analysis from the Desouza et al., (2004) study suggests that more experienced teachers in terms of years taught had the highest correlation of efficacy and outcome expectancy measures. Findings from Swars et al., (2007) found no relationship among MKT scores, teaching efficacy beliefs, and teaching outcome expectancy beliefs amongst pre-service elementary teachers, however, the findings are based on data collected from early elementary school teachers many of whom do not teach mathematics solely. The present study will assess the KAT, teaching efficacy, and outcome expectancy of middle and high school teachers who are trained to specifically teach mathematics thus it is hypothesized that KAT scores will predict teaching efficacy and outcome expectancy beliefs.

H_3: In prior research, early elementary teacher's mathematical knowledge for teaching (MKT) was not significantly correlated with teacher characteristics such as years of experience (Hill, Rowan, & Ball, 2005; Hill & Lubienski, 2007). However, the researcher hypothesizes that teacher characteristics and knowledge of algebra for teaching (KAT) among middle and high school teachers will be positively correlated. It is also hypothesized that KAT will mediate teaching characteristics and teaching efficacy, based on Swars, Smith, Smith, and Hart (2009) discovering a positive correlation between pre-service teachers' personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching (MKT) scores. Swars et al., (2007) found no relationships between learning mathematics for teaching scores (which measured specific mathematics knowledge required for teaching) of elementary school teachers, and mathematics teaching outcome expectancy scores.

Definition of Terms

Several key concepts and terms used throughout this study require definitions to clarify their meaning. The concepts and terms include:

Teaching efficacy: The teacher's belief in his or her capability to organize and execute courses of action required to successfully accomplish a specific teaching task in a particular context. (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998).

Outcome expectancy: A person's estimate that a given behavior will lead to certain outcomes (Bandura, 1977). A teacher's belief that effective teaching can bring about student learning (Swars, Hart, Smith, Smith, & Tolar, 2007).

Pedagogical content knowledge: Knowledge that transcends beyond knowledge of subject matter to the dimension of subject matter knowledge for teaching (Shulman, 1986).

Mathematics knowledge for teaching: Mathematics knowledge for teaching includes four subdomains; common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). Common content knowledge is the mathematical knowledge and skill used in settings other than teaching, specialized content knowledge is the mathematical knowledge and skill unique to teaching, knowledge of content and students is knowledge that combines knowing about students and knowing about mathematics, and knowledge of content and teaching combines knowing about teaching and knowing about mathematics (Ball, Thames and Phelps, 2008).

Knowledge of algebra for teaching: Based on three categories of knowledge key to effective teaching of algebra that reflect three perspectives on what teachers need to know and what they should be taught in their teacher preparation programs: knowledge of school algebra, knowledge of advanced mathematics, and knowledge of algebra for teaching. Knowledge of school algebra - knowing what they will teach; knowledge of advanced mathematics - knowing more advanced mathematics that is relevant to what they will teach; knowledge of algebra for teaching- knowing mathematics that is particularly relevant for teaching and would not typically be taught in undergraduate mathematics courses (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012).

Chapter II

Literature Review

Many theories have proposed to explain predictors of teaching efficacy and outcome expectancy as well as the pedagogical and content knowledge necessary for teaching mathematics using various questionnaires and survey instruments. Although the literature covers a wide variety of such theories, I will provide an overview of Albert Bandura's social cognitive theory, which includes teaching efficacy and outcome expectancy, an overview of pedagogical content knowledge, subject matter knowledge, mathematics knowledge for teaching, and knowledge of algebra for teaching with focus on their application to mathematics teachers.

Social Cognitive Theory

Social cognitive theory (earlier called social learning theory) proposed by Albert Bandura originally developed as a retort to behaviorism which is centered around the stimulus-response approach to human development. Alternatively, social cognitive theory is based on the idea that individuals learn from observing others in social contexts, not solely from direct reinforcement as proposed by behaviorism. In social cognitive theory, learning is defined as an internal mental process that may or may not be reflected in immediate behavioral change (Bandura, 1986). One of the main tenets of this theory is the idea of reciprocal determinism which states that the individual, the environment, and behavior are all interacting to influence one's behavior. These interacting factors can be illustrated using an individual's perceived self-efficacy, or beliefs concerning one's capabilities to organize and implement actions necessary to learn or perform behaviors at designated levels (Bandura, 1997).

Perceived self-efficacy is a central process affecting one's sense of personal agency (Schunk, 2012) and among the mechanisms of personal agency, none is more pervasive than individual's beliefs about their capabilities to exercise control over events that affect their lives (Bandura, 1989). Self-efficacy beliefs function as an important set of proximal determinants of human motivation, affect, and action which in turn effect thought patterns that may be self-aiding or self-hindering (Bandura, 1989). Outcome expectations are an individual's personal beliefs about the anticipated outcomes their actions (Schunk & Zimmerman, 2006). Social cognitive theory contends that people generate outcome expectations about the likely consequences of given actions based on their personal experiences and observations of models (Bandura, 1986, 1997). Although social cognitive theory spans a wide range of topics such as observational learning, motivation, and self-regulated learning most research within the theory is on self-efficacy and outcome expectancy.

Teaching efficacy and outcome expectancy. Bandura's social cognitive theory and his construct of self-efficacy have been the nucleus for developing the teaching efficacy construct. According to Bandura (2006) self-efficacy is defined as one's perceived capability on a given task. With each task, the individual determines through cognitive processes (e.g. thinking, planning, self-evaluating) whether or not they believe they are equipped to succeed at a specific duty or assignment. An individual's efficacy is hypothesized to influence their choice of activities, effort expended, perseverance when difficulties are encountered, and skillful performance (Schunk, 1984). Efficacy is a future-oriented judgment that has to do with perceptions of competence rather than actual levels of competence (Hoy & Spero, 2005). Perceived efficacy concerns judgments of how well one can execute tasks required (Bandura, 1982) to address various circumstances. In relation to teachers, efficacy could be influenced by one's beliefs associated with successful course instruction.

Social cognitive theory contends that people form outcome expectations about the likely consequences of given actions based on personal experiences and observations of models (Bandura, 1986, 1997). An individual's outcome expectations are personal beliefs about the anticipated outcomes of their actions (Schunk & Zimmerman, 2006). These outcome expectations can refer to both external and internal outcomes. Despite Bandura's social cognitive theory highlighting self-efficacy and outcome expectations as being conceptually distinct, they often are related. Schunk (2012) contends that individuals who typically perform well have confidence in their capabilities or efficacy beliefs and expect (and usually receive) positive outcomes for their efforts.

Tschannen-Moran and Woolfolk Hoy (2001) designate that a teacher's efficacy belief is a judgment of his or her teaching capabilities. Efficacy beliefs in the workplace are not static and reflect a lifelong process of development influenced by personal attributes and interpretation of environmental circumstances (Klassen & Chiu, 2010). A considerable portion of a teacher's efficacy beliefs can be influenced by the teaching environment, which may include the subject matter taught, workplace climate, and prior teaching experiences. According to Schunk (1984), educational practices have an important influence on efficacy. Such practices can be learned through college courses, teacher training programs, or on the job experiences. When evaluating efficacy, consideration must be taken for the context in which the efficacy measure is referring to. Consequently, in terms of evaluating teaching efficacy, we need to include the teaching task, context, and the weaknesses as well as the qualifications of the teacher with respect to the required task (Poulou, 2007). Current teacher efficacy scales vary in design to assess perceived teaching capability on a general level as well as a task specific level (e.g., mathematics teaching efficacy). The sources of a teacher's efficacy are critical in influencing both teacher behaviors and student learning among secondary mathematics teachers (Tschannen-Moran & Woolfolk Hoy, 2001). According to Bandura (1986), there are four major sources that influence an individual's efficacy perceptions. These sources were identified as mastery experiences, vicarious learning, social persuasion, and physiological states. These four sources are seen as interacting to effect efficacy judgments.

Sources of teaching efficacy. Bandura (1997) argued that mastery experiences in teaching, whether successful or unsuccessful, have the greatest influence on beliefs. However, Leader-Janssen and Rankin-Erickson (2013) contradict the assertion that unsuccessful experiences of teaching will contribute to mastery experiences, it is noted that proficiency of performance not inefficiency, creates a mastery experience. From these mastery experiences, teachers will shape future efficacy beliefs. Mastery experience is hypothesized to work in a cyclical nature where higher efficacy tends to lead to greater effort and persistence, (Leader-Janssen and Rankin-Erickson, 2013) which leads to better performance and teaching ability, which in turn leads to higher efficacy.

Vicarious experiences involve observing others succeed at a task, which may raise the belief that the observer could also succeed in performing the task (Swan, Wolf, & Cano, 2011). In teaching, this may occur when one indirectly experiences an individual teaching students (e.g. observing a model teacher). According to Poulou (2007) modeling serves as an effective tool for promoting a teacher's sense of efficacy. Khourey-Bowers & Simonis (2004) assert that the ultimate vicarious experience for teachers is to observe and critique others as they teach. Therefore, being exposed to strong teacher models may prove beneficial to a teacher's efficacy for teaching mathematics.

Social persuasion is said to involve the encouragement or discouragement that one receives from others for engaging in particular activities (Lent, 1996). Bandura (1997) suggested that positive changes to efficacy come about only through preemptive pressing feedback, which dissolves any pre-existing negative beliefs about one's abilities. This can come in the form of feedback from a mentor teacher, administrators, parents, encouragement from colleagues, and students.

According to Bandura (1997) physiological states can influence efficacy in that the level of arousal, either anxiety or excitement, can enhance one's efficacy. For a teacher, physiological states and reactions can include the pleasant or unpleasant emotional and physical sensations that one experiences while performing particular tasks (Lent, 1996) that are associated with teaching such as lecturing, lesson planning, and grading papers.

Early Teaching Efficacy Research. As a result of self-efficacy scales administered to teachers being associated with positive teaching behaviors and positive student outcomes, research on the teacher efficacy construct began to blossom in the 1980's. Gibson and Brown (1982) analyzed differences in teacher efficacy and personal teaching efficacy patterns in relation to levels of professional training and teaching experience. Researchers administered the Teacher Efficacy Scale (TES) to pre-service teachers at different stages of training and in-service teachers with varying years of experience, in which the results revealed that pre-service teachers with the least amount of training demonstrated the least personal teaching efficacy, indicating that they were not confident about their teaching skills. In the following years, as an effort to address the context specificity and measurement problems within the construct of teacher efficacy Ashton, Buhr, and Crocker (1984) developed a series of vignettes which demonstrated scenarios that a teacher might encounter on the job. Using these vignettes, teachers were asked to make judgments regarding their effectiveness in addressing prospective situations that may occur in the classroom such as teaching difficult or unmotivated students. Subsequently, in their study Teacher Efficacy: A Construct Validation, Gibson and Dembo (1984) took excerpts of Bandura's self-efficacy construct in order to hone in on and validate the teaching efficacy construct as a whole. In total, there were three phases in which this validation occurred; factor analysis, multitrait-multimethod analysis, and classroom observation. Teacher efficacy in the study was assessed using the 30-item (TES) Teacher Efficacy Scale (Gibson and Dembo, 1984) using the Likert style responses.

In phase one of the study, researchers (Gibson and Dembo, 1984) asked 208 elementary school teachers selected from 13 elementary schools to complete the TES survey. Principal factoring analysis was used to analyze the underlying factor structure of teacher responses to the TES. According to results from the study, a high score on the personal teaching efficacy subscale indicated high efficacy, whereas a low score on the teaching efficacy subscale indicated high outcome expectancy. In this study, both teaching efficacy and outcome expectancy are based on Bandura's proposal that one's behavior is determined by both a general outcome expectancy (belief that behavior will lead to desirable outcomes) as well as a sense of efficacy (belief that one has the requisite skills to bring about the outcome). These two factors were extracted based on the theoretical notion of Bandura's two-factor model of self-efficacy and Catell's scree test. An oblique rotation was used to compare item loadings and degree of correlations between factors, which revealed that the two factors were only moderately correlated (r = -.19) suggesting that a teacher's efficacy and outcome expectancy represent related, but comparatively independent constructs. One tailed t-tests were used to analyze the difference between high efficacy and low efficacy teachers, and the differences between high and low efficacy teachers were found in time spent in whole class versus small group instruction, teacher use of criticism, and teacher lack of persistence in failure situations.

Dembo and Gibson (1985) further studied the environmental effects of teacher efficacy, identifying the construct as an important factor in school improvement. Researchers found that teacher's classroom behavior varied significantly depending upon a teacher's efficacy. The identified behaviors examined included instruction, classroom organization, and teacher feedback provided to students who were experiencing difficulty. Specifically, significant differences were observed in teacher feedback patterns following a student's incorrect response. Teachers identified as having low-efficacy were more likely to give the student an answer, divert and ask another student, or allow another student to shout out an answer before the student who originally provided the incorrect response was given time to generate a correct response. Teachers identified as having high-efficacy demonstrated that they were more effective in leading students to correct responses through their questioning, without simply moving on. According to

Dembo and Gibson (1985) the problem of identifying such antecedents of a teacher's efficacy and developing ways to enhance teachers' sense of efficacy is critical.

Recent trends in teaching efficacy research. Researchers and theorists agree that teaching efficacy should be both context and subject matter specific. In terms of the evaluation of teaching efficacy, it is necessary to include the teaching task and the teaching context (Poulou, 2007). Should we simply focus on teaching, or should we narrow the specificity to teaching math, or even further narrow the subject matter to teaching algebra? What remains unclear is the appropriate level of specificity (Tschannen-Moran & Woolfolk Hoy, 2001) needed to properly measure teaching efficacy.

In order to address the lack of content specificity with earlier measures of teaching efficacy, subject matter modifications developed by Riggs and Enochs (1990) based on the Dembo and Gibson (1985) method were created to measure teachers' efficacy toward teaching science. The Riggs & Enochs (1990) instrument titled Science Teaching Efficacy Beliefs Instrument (STEBI) identified two uncorrelated factors within STEBI, which they named personal science teaching efficacy (PTSE) and science teaching outcome expectancy (STOE). This instrument was subsequently modified to become the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) by Enochs, Smith, and Huinker (2000) in efforts to measure teachers' efficacy toward teaching mathematics. An analysis of reliability produced an alpha coefficient of 0.88 for the personal mathematics teaching efficacy (PMTE) scale and an alpha coefficient of 0.75 for the mathematics teaching outcome expectancy (MTOE) scale. Additionally, a confirmatory factor analysis indicated that PMTE and MTOE subscales are independent.

Therefore, the MTEBI appears to be valid and reliable in assessing mathematics teaching efficacy and outcome expectancy.

Mathematics Teaching Efficacy Beliefs

Considerable research has been conducted using the Mathematics Teaching Efficacy Beliefs Instrument. Researchers who used the MTEBI to assess mathematics teacher's teaching efficacy and outcome expectancy beliefs include Swars (2005); Utley, Bryant, and Moseley (2005); Gresham (2008); Swars, Daane, and Giesen (2006); Cakiroglu (2008); Swars, Smith, Smith, and Hart (2009); Evans (2011); Brown (2012); and Kim, Sinh, and Mitchell (2014). Most of the aforementioned researchers administered the instrument to elementary mathematics teachers. Although there are numerous studies which examine mathematics teacher's teaching efficacy and outcome expectancy beliefs, there is a dearth of research which specifically examine the beliefs of mathematics teachers at the secondary level.

Many researchers examined the effects on mathematics teacher's efficacy as participants were enrolled in mathematics methods courses specifically designed for students studying to become mathematics teachers. Swars (2005) administered the MTEBI to four elementary pre-service teachers who had just completed a 3 semester hour undergraduate mathematics methods course. Within this course, participants received explicit instruction on a variety of effective mathematics teaching strategies such as using manipulatives and games to aid instruction. Participants also completed a clinical experience component within their undergraduate course. Researchers highlighted that mathematics instructional strategies as well as past experiences with mathematics and their influences upon perceptions of teaching effectiveness were associated with the mathematics teacher's efficacy in the study. Furthermore, Utley, Bryant, and Moseley (2005) examined approximately 60 pre-service teachers who were completing their final 9 months of their undergraduate studies. At this point in their undergraduate career, all of the participants had completed at least 4 mathematics content courses. Additionally, their coursework included a 12-week student teaching experience. As a part of their undergraduate program the participants were required to teach to their peers, tutor a child weekly in mathematics, and work extensively with manipulatives. Researchers conducted three administrations of the MTEBI survey during which data revealed that as the mathematics teacher progressed through the methods course, their mathematics teaching efficacy significantly increased. Swars, Smith, Smith, and Hart (2009) investigated the impact of a teacher preparation program on 24 pre-service teachers and found that the features of the teacher preparation program including mathematics courses and field experience, provided a context supporting teacher change regarding the teachers' mathematics pedagogical and teaching efficacy beliefs and specialized content knowledge for teaching mathematics. Evans (2011) examined the mathematical content knowledge secondary mathematics teachers had both before and after taking a mathematics methods course. The sample included 42 teaching fellows who were given a mathematics content test, attitudes toward mathematics questionnaire, and teaching efficacy questionnaire at the beginning and end of the semester. Findings revealed no statistically significant difference between pretest scores and posttest scores for teacher's personal mathematics teaching efficacy and mathematics teaching outcome expectancy which indicates there were no increases in teaching efficacy over the course of the semester.

Other researchers studied the relationship between teacher's mathematics efficacy beliefs and mathematics anxiety. When studying this relationship among 156 pre-service teachers, Gresham (2008) found a significant, negative relationship between mathematics teachers' efficacy and pre-service teachers' mathematics anxiety (r = -.475, $p<.05$). Swars, Daane, and Giesen (2006) examined 28 pre-service teachers who were completing a 3 credit hour mathematics methods course. Prior to taking the mathematics methods course, the participants had completed 9 hours of college mathematics courses with the most frequent coursework occurring in pre-calculus, algebra, and a mathematics for elementary teachers' course. Results indicated that the pre-service teachers with lower mathematics anxiety generally had higher mathematics teacher efficacy, and the pre-service teachers with higher mathematics anxiety generally had lower mathematics teacher efficacy.

In an international study, Cakiroglu (2008) compared the teaching efficacy of Turkish and Unites States pre-service teachers, in which the Turkish sample included 141 pre-service elementary teachers and the American sample included 104 pre-service elementary teachers. Results from the study indicated that pre-service teachers in Turkey tend to have a stronger belief that teaching can influence student learning when compared with pre-service teachers in the United States. However, a similar difference was not observed for personal mathematics teaching efficacy.

Brown (2012) investigated the characteristics of pre-service teachers who received their teacher education training in non-traditional settings in relation to their efficacy beliefs about mathematics. These characteristics included their ages, high-stakes math failures, lower division mathematics history, and math methods course performance. Results revealed that pre-service teachers' ages, mathematics history, and mathematics methods course performance, had a significant relationship with their math teaching efficacy beliefs.

Most recently, Kim, Sinh, and Mitchell (2014) studied in-service elementary school teacher's mathematics teaching efficacy. The participants in their study were 82% female; which was representative of the gender proportion in South Korea where the study took place, 80% held an elementary teacher certification, 79% held a bachelor's degree in elementary education, and 48% did not have an academic degree related to elementary mathematics education. There were significant differences among groups of teachers mean scores based on their years of teaching experience, levels of certification, and obtained degree relating to mathematics education. Additionally, there was a group of participants holding a master's degree related to mathematics education who had a statistically significantly higher mean of MTEBI scores than did both the group of bachelor's degree-holding participants related to mathematics education and the groups of non-degree holders. With the exception of Evans (2011), there is a lack of research studies which examine the mathematics teaching efficacy beliefs of secondary mathematics teachers. In the present study, MTEBI will be used amongst this understudied group of teachers as the questionnaire items have an established reliability and validity, and have been used in multiple studies examining teaching efficacy.

Outcome expectancy in teaching. Bandura's theory has been used frequently in teacher efficacy studies; with two dimensions of teacher efficacy identified as independent measures: teaching efficacy and outcome efficacy (Hoy & Spero, 2005). According to Swars, Hart, Smith, Smith, & Tolar; (2007) teaching outcome expectancy, is a teacher's belief that effective teaching can bring about student learning regardless of external factors such as home environment, family background, and parental influences. A teacher's outcome expectancy reveals the teacher's perception of the students' ability to learn from his or her direct teaching (Newton, Leonard, Evans, & Eastburn, 2012). Although a teacher may have high mathematics teaching efficacy, this does not guarantee that the teacher believes that they can influence a student's learning. Tschannen-Moran, Woolfolk Hoy, and Hoy (1998) assert that efficacy expectations provide individuals with a way to decide whether they have the ability to perform the required task at the desired level of competency, while outcome expectancy provides individuals a way to decide if they have accomplished a task at a desired level. The differences between teaching efficacy and outcome expectancy has to do with the extent to which the teacher perceives he or she has control of the situation (Buss, 2010). Teacher's levels of efficacy and outcome expectancies may vary depending upon the domain and the task.

In their 2007 study, Swars and colleagues examined pre-service elementary teacher's beliefs about mathematics teaching. The participants were enrolled in a two-year undergraduate teacher education program while being concurrently enrolled in two mathematics methods courses. Participants were surveyed a total of four times throughout this program. One of the findings showed that mathematics teaching outcome expectancy subscale scores significantly increased during the teacher preparation program. This indicated that as teachers were becoming familiar with the pedagogical foundations of teaching, their outcome expectancy advanced. If teachers of all experience levels were surveyed, as opposed to solely pre-service teachers, it may reveal that there is an increase in outcome expectancy as teachers feel more prepared and progress in their career, or as they engage in more mastery experiences.

Although prospective teachers may expect to teach math effectively, they may not feel they can exercise much control over whether students learn the information or are able to use it in a substantive way after instruction (Bandura, 1997; Tschannen-Moran et al., 1998). As many teachers are faced with the challenge of making math relevant to the real world, outcome expectations are affected. Beliefs about expectancy, ability, perceptions of task value, and perceived task difficulty influence the choices that new and experienced teachers make about their work (Watt & Richardson, 2007), which in turn influence decisions on how teachers will present the mathematical content to students. The beliefs that individuals hold about their abilities and about the outcome of their efforts powerfully influence the ways in which they will perform (Pajares, 1996) a given task. Such behaviors are only enacted when people not only expect specific behavior to result in desirable outcomes, but they also believe in their own ability to perform the behaviors (Enochs, Smith, & Huinker, 2000). Thus, teachers who demonstrate low outcome expectancy may opt out of certain activities due to a lack of strong ability beliefs.

Using the Science Teaching Efficacy Beliefs Instrument (STEBI) Desouza, Boone, and Yolmaz (2004) found that the science-teaching efficacy of elementary and middle school pre-service teachers showed positive correlations among efficacy and outcome expectancy. A strong correlation (r = .627) was obtained when considering middle school teachers' efficacy and outcome expectancy measures; elementary school teachers' measures of efficacy and outcome expectancy were also significantly correlated at (r = .399). This data suggests that teacher's efficacy beliefs and outcome efficacy are related, which is fairly different from prior findings of other researchers. Additionally, it was found that the more years a science teacher taught, the lower their outcome expectancy (Desouza et al., 2004) suggesting that as teachers gain more experience they may begin to feel as though they have a decreased influence over their student's outcomes.

Teaching Knowledge

What a teacher needs to know in order to demonstrate competency to teach math is highly debated and generally vague depending on who you ask. This section will review prior research and theories based on what is considered necessary knowledge for teaching.

Pedagogical content knowledge. To have pedagogical knowledge is to know how to teach, to have content knowledge is to know what to teach. Both are necessary components in demonstrating proficiency as a teacher. Pedagogical content knowledge encompasses both the knowledge of representations of subject matter (content knowledge); understanding of students' conceptions of the subject, the learning, and teaching implications that were associated with the specific subject matter; and general pedagogical knowledge (or teaching strategies); curriculum knowledge; knowledge of educational contexts; and knowledge of the purposes of education (Shulman, 1986). According to Shulman (1986), pedagogical content knowledge extends beyond general subject matter knowledge toward the dimension of subject matter knowledge for teaching. In studying the role of pedagogical content knowledge in teaching mathematics, Sherin, (2002) found that teacher's pedagogical content knowledge was enhanced as they negotiated among their own knowledge of student thinking, mathematics, and the lesson. Additionally, Ebby (2000) found that pre-service teachers' knowledge and views of mathematics were expanded as they made connections between their own experiences as learners and as teachers. Throughout the past 30 years, researchers of mathematics teacher education have expanded the notion of pedagogical content knowledge toward more fine-tuned, content specific measures meant to assess teacher's specific mathematics knowledge.

Mathematics knowledge for teaching. According to Ball, Hill, and Bass (2005) subject matter knowledge for teaching mathematics is defined as being composed of two key elements: 1) common knowledge of mathematics that any well-educated adult should have and 2) mathematical knowledge that is specialized to the work of teaching and that only teachers need know. Examples of skills that only mathematics teachers need to know include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs.

Building upon Shulman's work, researchers (Hill et al., 2004; Ball et al., 2005; Ball et al., 2008) developed mathematical knowledge for teaching. MKT is believed to be a blend of general knowledge of mathematics and mathematical knowledge specialized for teaching (Ball, Hill, and Bass, 2005). Within their model, researchers proposed two domains, subject matter knowledge and pedagogical content knowledge. Subject matter knowledge is proposed to include 3 subdomains: common content knowledge, horizon content knowledge, and specialized content knowledge. Pedagogical content knowledge also pinpoints 3 subdomains: knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum. A multiple-choice test specifically designed to measure teachers' mathematics knowledge for teaching at the elementary level was created by researchers at the University of Michigan as part of the Learning Mathematics for Teaching study (Hill et al., 2004; Ball et al., 2005; Ball et al., 2008) which later developed into measures for teachers at the middle level. Their work has shown that mathematical knowledge for teaching, as measured by questions developed specifically for elementary and middle school mathematics teachers, significantly correlates with effective mathematics teaching, measured by Kindergarten-8th grade students' scores on standardized tests.

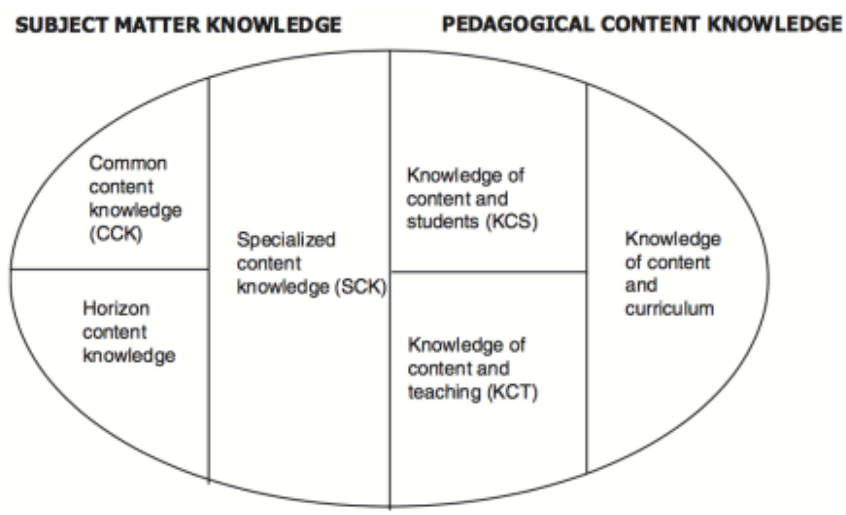


Figure 1. Model of MKT Framework (Ball et al., 2008, p. 403)

Beyond pure subject matter knowledge, a teacher needs to know how to teach mathematics including knowledge of how to represent mathematical topics and ideas in a way that children can understand, knowledge of mathematics curriculum materials and resources, and knowledge of how to organize and manage a mathematics classroom (Shulman, 1986). Although many states address content knowledge through requiring teachers to get a certification in a specific area (e.g. elementary math, middle school math, or high school math) to demonstrate proficiency, mathematics teachers should also know how to correctly calculate problems, use pictures or diagrams to represent mathematics concepts and procedures, provide explanations for common rules and mathematical procedures, and analyze students' solutions and explanations (Hill et al., 2005). The research on MKT is gradually developing, as MKT is a new construct that has blossomed over the last decade.

Hill et al., (2005) studied mathematics knowledge for teaching among a sample of 334 first-grade and 365 third-grade teachers to determine whether a teacher's mathematics knowledge for teaching effects student achievement. Results indicated that elementary teachers content knowledge for teaching was a significant predictor of academic gains for both 1st and 3rd grade students. In a study of first year teachers, (Rockoff, Jacob, Kane, & Staiger, 2008) examined teaching specific content knowledge, cognitive ability, personality traits, feelings of self-efficacy, and scores on a teacher selection instrument with intentions to identify predictors of teacher effectiveness. Results of the study found specialized mathematics knowledge to be a better predictor of student mathematics outcomes than general cognitive ability. In a sample of German 10th grade teachers (Baumert et al., 2010) investigated the significance of teachers' content knowledge and pedagogical content knowledge for high-quality instruction in relation to student progress in secondary-level mathematics. Results indicated that teachers pedagogical content knowledge was more predictive of student achievement outcomes than content knowledge. Furthermore, Hill (2010) explored elementary teacher's mathematics knowledge for teaching and teaching characteristics in which results indicated a significant relationship between MKT scores and specific teacher characteristics including leadership activities and self-reported college level mathematics preparation. Copur-Gencturk (2015) conducted a longitudinal study which examined the mathematics knowledge for teaching and instruction of twenty-one Kindergarten-8th grade teachers enrolled in a master's degree program over a three-year period. Using multilevel multivariate growth modeling, results of the study indicated that expansions in teacher's mathematical knowledge predicted change in their instructional practices including the quality of their inquiry-based lesson design, mathematical sense-making agenda, and classroom climate.

Findings from studies that evaluated teacher's mathematics knowledge for teaching provide an indication of the importance of utilizing an instrument that is able to simultaneously assess content specific knowledge and knowledge specific to teaching. With exception of the Baumert et al., (2010) study, little research has been conducted examining teaching knowledge among teachers of secondary mathematics students. Subsequently, development of a content specific measure for knowledge of algebra teaching was underway.

Knowledge of algebra for teaching. According to the National Council for Teaching Mathematics (2014b), all students should have access to algebra within a Pre-Kindergarten -12th grade mathematics curriculum, including opportunities to generalize, model, and analyze situations that are purely mathematical and ones that arise in real-world phenomena. Algebraic concepts, procedures, and applications should be integrated across grade levels for consistency throughout the curriculum. Floden & McCrory (2007) and McCrory et al., (2012) developed a theory about the mathematical knowledge needed for effective algebra teaching derived from the early notion of the pedagogical content knowledge theory popularized by Shulman (1986), and the recent construct of mathematical knowledge for teaching by (Hill et al., 2004; Ball et al., 2005). This theory was subsequently used as the basis for developing an assessment of algebra teachers' knowledge, this assessment was called knowledge of algebra for teaching (KAT).

The three dimensions of KAT include school knowledge, advanced knowledge, and teaching knowledge. According to McCrory et al., (2012) these dimensions are defined as knowing what teachers will teach (school knowledge of algebra); knowing more advanced mathematics that is relevant to what teachers will teach (knowledge of advanced mathematics); and knowing mathematics that is particularly relevant for teaching and would not typically be taught in undergraduate mathematics courses (mathematics for teaching knowledge).

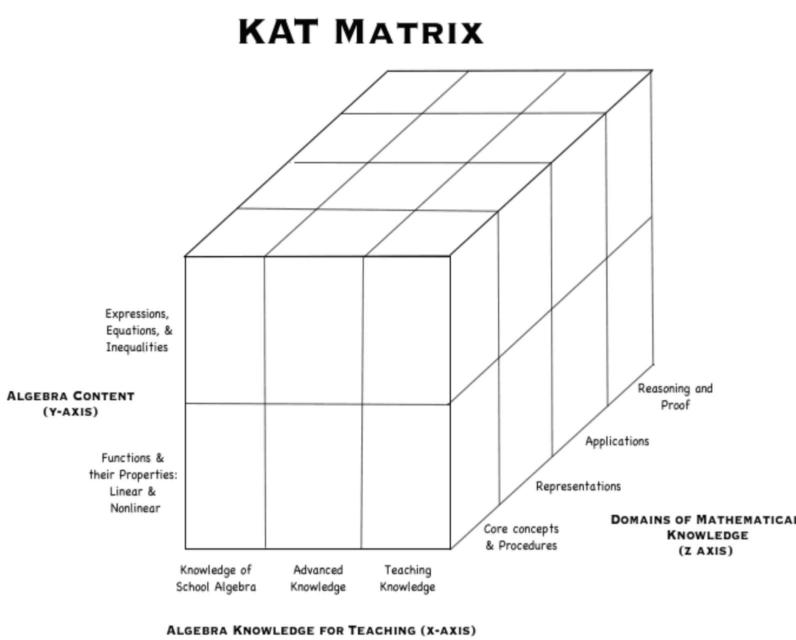


Figure 2. Model of KAT Framework Matrix (Floden & McCrory, 2007).

School knowledge of algebra. Algebraic proficiency comes from a report by the RAND Mathematics Study Panel (2002) which identified what teaching and learning algebra concepts in Kindergarten through 12th grade consists of.

- The ability to work flexibly and meaningfully with formulas or algebraic relations and use them to represent situations, to manipulate them, and to solve the equations they represent.
- A structural understanding of the basic operations of arithmetic and of the notational representations of numbers and mathematical operations (for example, place value, fraction notation, exponentiation).
- A robust understanding of the notion of function, including representing functions (for example, tabular, analytic, and graphical forms); having a good repertoire of the basic functions (linear and quadratic polynomials, and exponential, rational, and trigonometric functions); and using functions to study the change of one quantity in relation to another.
- Knowing how to identify and name significant variables to model quantitative contexts; recognizing patterns; and using symbols, formulas, and functions to represent those contexts.

Knowledge of advanced mathematics. There are ranges of advanced mathematics topics that are recognized by researchers as providing a deeper understanding of school algebra concepts. These topics are calculus, linear algebra, number theory, abstract algebra, real and complex analysis, and mathematical modeling (McCrory et al., 2012).

Mathematics for teaching knowledge. According to McCrory et al., (2012) mathematics for teaching knowledge is defined as mathematics that is useful in teaching, but is not typically taught in conventional mathematics classes. The focus is not on how the teacher learns mathematics, but rather the focus is on mathematical ideas that teachers know and use that are not needed in other mathematical applications and other contexts. This includes ways of thinking about mathematics and interpreting students' mathematical language, which is similar prior researcher's (Shulman, 1986; Hill et al., 2005; Ball, Hill, and Bass, 2005) definition of specialized content knowledge. This form of knowledge can be acquired through various means including field experiences in a mathematics teacher preparation program, on the job training, mathematics methods courses, and professional development opportunities.

McCrory et al., (2012) also assert that the ability to decompress, trim, and bridge is necessary for algebra teachers. Topics that should be decompressed include solving equations and systems of equations, simplifying expressions, and demonstrating multiple representations. Throughout a teacher's career, one can encounter a class of students with individual learning differences. Many of these learners may perform on grade level and there are others who may perform below grade level. In these instances, the ability to utilize trimming is necessary when faced with the challenge of teaching an academically diverse group of students. Trimming or editing the mathematical content may be useful to teachers in efforts to reflect students' current level of sophistication while treating the mathematics with integrity (McCrory et al., 2012). Trimming can include adapting a textbook's treatment of an idea, modifying textbook problems, revising students' questions or comments, responding to student's thoughts about particular mathematical ideas, or creating new examples and problems (McCrory et al., 2012). Lastly, bridging includes understanding the intersection of algebraic concepts within algebra and with other mathematical domains (McCrory et al., 2012).

Item response theory. Two parallel assessment forms were constructed to measure the three dimensions of a teacher's knowledge for teaching algebra; school knowledge, advanced knowledge, and teaching knowledge. Following the construction of the assessment, researchers conducted a validation study using responses from 1170 pre-service and in-service mathematics teachers to gather evidence about construct validity. In the validity study, both forms had reliability of $\alpha = .84$. An item response theory analysis was used to calibrate and evaluate KAT test items on each of three dimensions: school knowledge, advanced knowledge, and teaching knowledge. Researchers created a basis for computing scale scores for overall test performance in which test items were rescaled so that the ability scores of the sample in the validity study had a mean of 50 and standard deviation of 10. For the subscale scores, a separate rescaling was done so that the ability scores for each subscale also had a mean of 50 and a standard deviation of 10 (Knowledge of Algebra for Teaching Project, 2009).

Importance of assessing knowledge of algebra for teaching. As the Common Core State standards (adopted by all but 8 states in the United States), and other individual state standards become more rigorous, an increasing number of teachers who are accustomed to teaching at a certain level of rigor and teaching a content to a particular level of proficiency, are needing to revise their teaching practices and adjust their previous conceptions on what is required to teach algebra. Although the term pedagogical content knowledge (PCK) has been widely used over the past 30 years, and mathematics knowledge for teaching (MKT) has been used in recent years among elementary and middle school mathematics teachers, these are constructs that merely scratch the surface of identifying what knowledge is necessary for teaching algebra.

The absence of measures of teacher knowledge that would capture the aspects of advanced and practice-based mathematics on which teachers would likely draw for effective algebra instruction is evident (McCrory, et al., 2012). Knowledge of algebra for teaching, although in its infancy, is important to study amongst secondary mathematics teachers as teaching algebra involves more than just determining the answer to a complex problem. Skillful math teaching requires being able to size up the source of a mathematical error (Ball, Thames, & Phelps, 2008), which teachers must do rapidly, because in a fast paced classroom environment every minute of instruction counts. According to Peterson, Fennema, Carpenter, and Loef (1989), teacher's beliefs, content knowledge, judgments, thoughts, and decisions have a profound effect on the way teachers teach, as well as on students' learning in their classrooms. As knowledge of algebra for teaching beliefs are assessed, questions of not only what teacher's know, but how a teacher will deliver knowledge to students will be addressed.

A teacher's ability to convey mathematics knowledge properly, specifically in algebra, is an increasing concern as students have questions about misconceptions and underlying mathematical processes. Knowing mathematics well is central to a teacher's ability to use instructional materials wisely, assess students' progress, and make sound judgments about presentation, emphasis, and sequencing (Ball et al., 2005). Finding a correct answer to a mathematical problem is only part of the problem of teacher knowledge; teacher's also need to be able to apply their knowledge in practical situations, responding quickly and accurately to student thinking, unexpected mathematics, and to materials that may not be appropriate for the circumstances of the classroom (McCrory et al., 2012). Teacher's knowledge of mathematics will support the teacher's explanations, demonstrations, diagnosis of misconceptions, acceptance of children's own methods, and curriculum decisions (Ernest, 1989). Mathematics teachers are more likely to develop and maintain perceptions of efficacy grounded in their level of preparation to teach (Ross et al., 1996).

Having the required knowledge to teach algebra is not only a matter of having a college degree in mathematics, or taking mathematics coursework, one must know how to teach in order to be effective. If a teacher cannot recognize mathematical errors, answer students' questions, and provide alternative methods for explanation, this may cause confusion for the learner, disappointment for the teacher, and may ultimately lead to ineffective teaching. It is this integration of content and pedagogy that is critical in the success of a teacher and the reason why assessing teacher's knowledge of algebra for teaching, no matter the results, will enrich the body of research within the KAT construct.

Teaching strategies for improving algebra knowledge. The Institute of Educational Sciences (2015) in partnership with the United States Department of Education published a list of recommended teaching strategies for improving algebra knowledge in middle and high school students. These recommendations parallel with the focus of the Knowledge of Algebra for Teaching study in assessing how a teacher would respond to various scenarios that arise within the context of teaching algebra. Recommendations from the Institute of Educational Sciences (IES) and required actions on behalf of teachers are as follows.

Recommendation: Use solved problems to engage students in analyzing algebraic reasoning and strategies. Actions: 1. Have students discuss solved problem structures and solutions to make connections among strategies and reasoning. 2. Select solved problems that reflect the lesson's instructional aim, including problems that illustrate common errors. 3. Use whole-class discussions, small-group work, and independent practice activities to introduce, elaborate on, and practice working with solved problems.

Recommendation: Teach students to use the structure of algebraic representations. Actions: 1. Promote the use of language that reflects mathematical structure. 2. Encourage students to use reflective questioning to notice structure as they solve problems. 3.Teach students that different algebraic representations can convey different information about an algebra problem.

Recommendation: Teach students to intentionally choose from alternative algebraic strategies when solving problems. Actions: 1. Teach students to recognize and generate strategies for solving problems. 2. Encourage students to articulate the reasoning behind their choice of strategy and the mathematical validity of their strategy when solving problems. 3. Have students evaluate and compare different strategies for solving problems.

In order to follow to the IES recommendations, secondary mathematics teachers must have a strong content knowledge of algebra and know what to teach, have a strong advanced knowledge of mathematics and use this knowledge to accelerate student's algebra knowledge, and have a strong pedagogical knowledge demonstrating they know how to teach, which mirror the focus of the KAT construct.

Teacher Characteristics

Teaching knowledge and teacher characteristics. Although there is little research examining the relation between the knowledge of algebra for teaching and teacher characteristics, there are prior studies which utilize pedagogical content knowledge (PCK) and mathematics knowledge for teaching (MKT) to assess this relationship. Per Shulman (1986), PCK is subject matter knowledge for teaching and one who has PCK should be able to formulate and represent the subject to make it comprehensible to others. MKT builds upon the concept of PCK to include teaching knowledge specifically related to mathematics teaching, and knowledge of algebra for teaching (KAT) further enhances specificity by measuring algebra teaching knowledge.

Measurement tasks associated with the MKT construct (Hill, Rowan, and Ball, 2005) evaluated teacher's proficiency at providing students with mathematical explanations and representations, as well as working with unusual solution methods. Using the measurement tasks, results of a correlation analysis conducted by Swars et al., (2007) revealed significant relationships between specialized content knowledge and pedagogical beliefs at the end of student teaching. Pre-service teachers' scores on the instrument showed a positive relation to curriculum and learner subscale scores, however there were no relationships between the scores and mathematics teaching efficacy and mathematics teaching outcome expectancy subscale scores.

In a comparison study, Depaepe et al., (2015) examined the rational number content knowledge and pedagogical content knowledge of prospective elementary teachers (1^{st}-6^{th} grade), trained as general classroom teachers and secondary teachers (9^{th}-12^{th} grade), trained as specialized mathematics teachers. Results of the study revealed a higher content knowledge scores for secondary teachers compared to elementary school teachers, but lower pedagogical content knowledge scores.

Hill and Lubienski (2007) further examined the relationship between the mathematical knowledge of Kindergarten-8th grade teachers in which grade level taught predicted content knowledge scores at a strongly significant level, indicating that the higher the grade level, the better that teacher performs on MKT. In an examination of middle school mathematics teachers, Hill (2007) found that teachers with more years of experience teaching mathematics had higher levels of teaching-specific mathematical knowledge. Becoming more knowledgeable about a particular subject increases efficacy (Schunk, 2012) for discussing the subject more accurately and completely. For the present study, which focuses on pre-service, former, and in-service secondary mathematics teacher's specific algebra knowledge needed for teaching, this may provide similar results.

Characteristics of high efficacy teachers. Ross (1994) suggested that teachers with higher levels of efficacy are more likely to learn and practice new approaches and strategies for teaching, use management techniques that enhance student autonomy and diminish teacher control, provide special assistance to low achieving students, build students' self-perceptions of their academic skills, set attainable goals, and persist in the face of student failure. According to Czernaik (1990) highly efficacious teachers were more likely to use reform-based teaching methods, such as inquiry-based and student-centered approaches, while teachers with low levels of efficacy used more teacher-directed methods, such as lecturing and textbook reading. In-service teachers, as well as pre-service teachers, who have high teacher efficacy use a greater variety of instructional strategies (Riggs & Enochs, 1990). Teachers with high levels of efficacy in instructional strategies implement effective classroom practices (Tschannen-Moran & Woolfolk Hoy, 2001). These observations indicate that teachers who demonstrate higher levels of efficacy are willing, eager, and persistent. A teacher with high levels of efficacy would be preferred in a mathematics classroom based on the outcomes thus far.

Years of experience. Bandura's social cognitive theory, based on the idea of the reciprocal interaction among the person, the environment, and their behavior, encompasses factors that can affect teacher's efficacy. Characteristics associated with teaching efficacy, outcome expectancy, and content knowledge include years of experience, teaching level, school type, gender, and ethnicity. According to Wolters and Daugherty (2007) more experienced teachers are likely to have further knowledge in the content they teach, have different attitudes about their students, and think and behave differently in the classroom when compared with their less experienced peers. Klassen and Chiu (2010) found that among Canadian in-service teachers' teaching grades K-12, years of experience and teaching grade level were linked to their instructional efficacy. Teachers' gender, years of experience, school type, teaching grade, and sources of stress were linked to their classroom management efficacy. The authors determined that teaching efficacy increased up to the midcareer point and began to fall at the late career point (Klassen and Chiu, 2010). Teaching efficacy beliefs increased as teacher's teaching experience reached 15 years and subsequently decreased afterward (Kim, Sihn, & Mitchell, 2014), indicating that the more experience one has teaching the lower the efficacy beliefs. On the other hand, Ghaith and Yaghi (1997) found that teachers' experience was negatively correlated with their sense of general teaching efficacy in a study conducted with 25 middle and high school teachers. In contrast, in a sample of secondary teachers in Canada with median years of experience between 11-19 years, Ross, Cousins, and Gallada (1996) found that a teachers' sense of efficacy was associated with higher years of teaching experience. Previous studies have shown inconsistent conclusions with the relation of years of experience to teaching efficacy.

According to Mulholland & Wallace and (2001), some of the most powerful influences on the development of teachers' sense of efficacy are experiences during student teaching and the induction year, which may imply that assessing the efficacy of a first year teacher is of high importance. Teachers' efficacy beliefs appear to affect the effort teachers invest in teaching, their level of aspiration, and the goals they set (Hoy & Spero, 2005) which may include their commitment to developing lessons, time spent planning, and seeking professional development opportunities.

Analysis from Desouza et al., (2004) suggests that the more experienced teachers in terms of years taught tended to have the highest correlation of efficacy and outcome expectancy measures ($r = 0.540$). Therefore, teachers who have been in their career longer may demonstrate a stronger mathematics teaching efficacy and outcome expectancy than less experienced teachers.

Teaching environment. Teachers make efficacy judgments, in part, by assessing the resources and constraints in specific teaching environments (Hoy & Spero, 2005). Due to the diversity across schools, this can lead to a variety of efficacy judgments. For example, one may feel inadequate in their career due to a lack of professional development in mathematics or having negative interactions with students, thus exhibiting lower levels of efficacy. Additionally, money needed to purchase mathematics resources such as textbooks, technology, and manipulatives can pose a problem within schools as individual schools are assigned a specific budget based on the population of their student body. In general, a school with large student enrollment may have a greater budget, which would allow for the purchase of additional mathematics resources, materials, and possibly hiring additional support staff such as a mathematics specialist. Comparatively, a school with low enrollment may have a lesser budget, thus, they are not able to purchase as many materials, resources, or hire support staff.

According to Title I, Part A of the Elementary and Secondary Education Act, financial assistance will be provided to local educational agencies and schools with high numbers or high percentages of children from low-income families to help ensure that all children meet challenging state academic standards (U.S. Department of Education, 2015). Thus, Title I schools may have an increased budget in order to train teachers and provide additional resources for the students in efforts to bridge the performance gap between low-income and high-income students. The teaching resources available and the quality of the facilities could all impact teachers' assessments about their ability beliefs (Hoy & Spero, 2005) to accomplish the tasks associated with mathematics teaching. Therefore, identifying environmental factors associated with teaching efficacy such as whether teachers have access to mathematics resources and professional development is necessary.

Teaching level. In a sample of Pre-Kindergarten -12th grade in-service teachers, Wolters and Daugherty (2007) found during between-subjects tests, that there was no indication of an effect of academic level and a significant but modest effect for teacher experience related to efficacy for instruction. Additionally, Wolters and Daugherty

(2007) found that teachers in higher grade levels (middle and high school teachers) reported lower efficacy than teachers in lower grade levels (elementary teachers), and an inverse relationship between teaching level and efficacy exists. In contrast, Guskey (1988) found no difference in grade level assignment or years of experience as a function of efficacy. However, the survey items included in the efficacy questionnaire are not identical to the MTEBI questionnaire that is used in the present study.

One of the limitations of current literature is the lack of specificity within the teaching efficacy (or outcome expectancy) construct. Many do not focus on mathematics content specifically designed for secondary mathematics teachers when examining teaching efficacy. The issue of teacher efficacy is a great concern in education, especially since the construct has proven to effect student achievement and student efficacy. Additionally, the shortage of specificity for mathematics teaching efficacy with respect to teachers across differing experience levels is lacking with many studies focusing solely on pre-service teachers.

Outcome expectancy and teacher characteristics. Teacher characteristics to be examined in the present study include, but are not limited to, years of experience, teaching level, access to a math specialist, certification, school type, gender, and ethnicity. Previous research using outcome expectancy measures are minimal and do not report identical characteristics or identical participants (i.e. secondary math teachers) as examined in the present study. However, in this study, implications of relations among results will be identified.

In their pilot study on the effect of a field biology professional development program on in-service teacher's efficacy, Khourey-Bowers and Simonis, (2004) found that overall results, as measured with the Science Teaching Efficacy Beliefs Instrument (STEBI), indicated that teachers showed statistically significant improvement overall in efficacy and in personal science teaching efficacy. There was also a marginally significant change in outcome expectancy based on findings of a paired t-test analysis. These results indicate that as teachers receive professional development in their subject matter (i.e. from a math specialist), their outcome expectancy will increase over time.

Also using the STEBI in their comparison and analysis of teacher efficacy beliefs and outcome expectancy, Angle and Moseley, (2009) related teachers STEBI responses to students' mean scores on a statewide biology examination. Researchers found no statistical significance between the mean difference among the two groups of teachers (non-proficient students and proficient students) on the personal science teaching efficacy subscale, indicating that personal science teaching efficacy was not statistically related to how a teacher's students scored on the biology test. The mean difference, however, on the teacher's outcome expectancy subscale scores demonstrated a statistically significant difference between the two groups of students, with the proficient group's teachers having a significantly higher outcome expectancy score than non-proficient group's teachers. This finding suggests that the biology test scores were related to the expectations that a teacher held for his or her students to learn biology. Information included in their study specified that participants were 59.7% female and 40.3% male; 32% of the teachers were between 46-55 years old; 65.8% earned a bachelor's degree, 33.2% earned a master's degree, 1% earned a doctorate degree; 85.2% earned their teaching certification through a traditional route and 14.8% earned their teaching certification through an alternative route. Although it was not directly reported in the results which individual groups had more proficient or non-proficient student groups, it is important to note that these characteristics may be an indicator of outcome expectancy scores.

When exploring the science-teaching efficacy and science-teaching outcome expectancy beliefs of secondary school teachers in the United Arab Emirates, researchers Hassan and Taraib, (2012) questioned the impact of teaching experience to outcome expectancy beliefs and the impact of gender to outcome expectancy beliefs. In regards to teaching experience, the proportion of variability in outcome expectancy accounted for by years of teaching experience was .03, indicating a low relationship between years of teaching experience and teachers' perceptions of their outcome expectancy beliefs. In relation to gender, the proportion of variability in outcome expectancy accounted for by the gender was .001, indicating a low relationship between gender and teachers' perceptions of their outcome expectancy beliefs.

Chapter III

Methodology

The present chapter describes the methodology and procedures used to determine the teaching efficacy, outcome expectancy, and knowledge of algebra for teaching of secondary mathematics teachers. This chapter was organized into the following sections: participants, sample sizes, instrumentation, data collection procedures, data analysis procedures, and summary. Methods and procedures presented were applied to (a) determine the differences between teaching efficacy beliefs among secondary mathematics teachers (b) differences between outcome expectancy beliefs among secondary mathematics teachers (c) examine the relationship between the teaching efficacy beliefs and variables such as years of teaching experience, age, grade level taught, and gender (d) examine the relationship between the outcome expectancy beliefs and variables such as years of teaching experience, age, grade level taught, and gender (e) assess teacher's performance on a knowledge of algebra for teaching test in comparison to their teaching efficacy and outcome expectancy.

Research Design

Participants. Participants in the study were required to meet two criteria: the participants must be at least 18 years of age and they must be pre-service (preparing for a career in teaching), in-service (currently in employed as a teacher), or former (previously employed as a teacher) middle or high school mathematics classroom teachers. The participants included 100 middle and high school mathematics educators. The participants consisted of undergraduate and master's students enrolled in various mathematics focused teacher education programs at various urban southwestern public universities. University participants may self-identify as pre-service, in-service, or former mathematics teachers. Participants also included current and former mathematics teachers from independent school districts, charter schools, and private schools.

Demographic data were collected on a variety of variables including ethnicity, highest level of education attained, teacher type, certification, and age. All reported items are based on (N=100). Participants identified as the following; gender: 20% male, 80% female; ethnicity: 75% white, 3% Hispanic or Latino, 13% Black or African American, 5% Pacific Islander, 4% other (for the purpose of the study the groups were consolidated into White 75% and Non-White 25%); highest level of education attained: 5% HS Diploma, 47% Bachelors, 44% Masters, and 4% Doctorate; teacher type: 9% pre-service, 83% in-service, and 8% former; age: minimum 20 years old, maximum 65 years old, mean 36.11 years old, standard deviation 11.58, years of teaching experience: minimum 0 years, maximum 40 years, mean 8.66 years; grade level taught or preparing to teach: 42% middle school teachers and 58% high school teachers.

Table 1
Demographics (N=100)

	Frequency	Percent (%)
Ethnicity		
White	75	75
Non-White	25	25
Highest Degree Attained		
High School Diploma	5	5
Bachelors Degree	47	47
Masters Degree	44	44
Doctorate Degree	4	4
Teaching Status		
Pre-Service	9	9
In-Service	83	83
Former	8	8
Teaching Certification		
Math	77	77
Non-Math	23	23
Age		
20-29 years old	35	35
30-39 years old	32	32
40-49 years old	14	14
50-59 years old	15	15
60-69 years old	4	4
Grade Level		
Middle School (grades 6-8)	42	42
High School (grades 9-12)	58	58
Years of Teaching Experience		
0 to 1 years	14	14
1.01 to 3 years	17	17
3.01 to 5 years	9	9
5.01 to 7 years	13	13
7.01 to 10 years	14	14
10.01 to 15 years	11	11
15.01 to 20 years	16	16
20.01 to 25 years	3	3
25 or more years	3	3

Quantitative data were collected from this sample to examine the relationship between mathematics teaching efficacy, outcome expectancy, and knowledge of algebra for teaching. Additional demographic data and results of this data collection and analysis can be found in Chapter IV.

Instrumentation

Items from each of the surveys described below were combined into a single instrument administered to participants in one session.

Demographic questionnaire. Participants answered open-ended, checklist, fill-in-the blank, and multiple-choice questions on the demographic questionnaire. Upon disaggregating response data, individual teacher characteristics were grouped based on frequencies and percentages (Table 1). The demographic questionnaire was coded as follows: 0- Non White, 1-White; 12-High School Diploma, 16-Bachelors Degree, 18-Masters Degree, 20-Doctorate Degree; 1-Pre-service middle or high school math teacher, 2-In-service (current) middle or high school math teacher, 3-Former middle or high school math teacher; 0-Non Math Certification, 1-Math Certification; 0-Middle School Teacher, 1-High School Teacher; it should be noted that both age and years of teaching experience are continuous variables which were only grouped for the purpose of displaying demographic statistics in Table 1.

Mathematics teaching efficacy beliefs. Mathematics Teaching Efficacy Beliefs Instrument (MTEBI); Enochs, Smith, & Huinker, (2000) items will be used in this study to measure mathematics teaching efficacy beliefs amongst middle and high school mathematics teachers. Each item on the 21-item survey includes five response categories on a standard Likert scale: strongly agree, agree, uncertain, disagree, and strongly disagree. The MTEBI is comprised of two subscales, personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). Of the 21 items, 13 items are on the PMTE subscale and 8 items are on the MTOE subscale. Reliability analysis produced an alpha coefficient of internal consistency (α=0.88) for the PMTE scale and an alpha coefficient of internal consistency (α=0.77) for the MTOE scale. A principal component analysis will be conducted to assess the underlying structure of the components within the reported MTEBI data.

Knowledge of algebra for teaching. Knowledge of Algebra for Teaching (McCrory, et al., 2012) is derived from the Mathematics Knowledge for Teaching (MKT) construct which is measured through the use of the Learning Mathematics for Teaching (LMT) instrument (Hill et al., 2004; Ball et al., 2005) in which items were created to measure the specialized knowledge of elementary and middle school mathematics teachers. Items for the LMT were written to focus on three topics in mathematics: number concepts & operations; geometry; and patterns, functions, & algebra with intended use amongst elementary and middle grade teachers. The researcher chose to use the recently developed and seldom used KAT instrument in order to solely assess algebra knowledge, as algebra represents a large portion of the foundational mathematical content covered in middle and high school grades which is deemed appropriate for the study participants. A 15-item selection with multiple-choice responses from Form A of the KAT instrument will be used which focuses on teacher's responses to algebra questions. Reliability analysis of Form A produced an alpha coefficient of internal consistency (α=.84). A principal component analysis will be conducted to assess the underlying structure of the components within the reported KAT data.

11. Mr. Matheson asked students to solve the following system of equations:

$$\begin{cases} 2x + y = 3 \\ 4x + 2y = 6 \end{cases}$$

Orlando wrote:

$$(-2)(2x + y) = 3\,(-2)$$

So
$$\begin{array}{r} -4x - 2y = -6 \\ 4x + 2y = 6 \\ \hline 0 = 0 \end{array}$$

This system doesn't have a solution.

Which of the following is true about Orlando's response?

A. Orlando's solution and reasoning are correct.

B. Orlando made an arithmetic error.

C. You cannot add equations.

D. Orlando drew the wrong conclusion from $0 = 0$.

E. None of the above.

Figure 3. Sample Knowledge of Algebra for Teaching Item

Data Collection Procedures

Administration of the questionnaire. Data was collected through quantitative measures using a combination of three self-report surveys. The survey questions were delivered in the following order: demographic questions, mathematics teaching efficacy beliefs instrument, and knowledge of algebra for teaching questions. All questionnaires were administered in the same order for all participants. The rationale for this is so that response bias, the bias that results from problems in the measurement process, can be reduced.

Incentives. Subjects who participated in the study and completed the full survey were allowed to enter an optional drawing to win a $25 Visa gift card. Entries for the drawing were collected using Google Forms and individual drawing entries were not attached to the survey responses. No other incentives were provided for participation.

Anonymity and confidentiality. Teachers invited to participate were current pre-service, in-service, or former middle or high school mathematics teachers. Participants were recruited through local school districts and teacher organizations. Consent was obtained by attaching an anonymous consent form to the beginning of the survey. The survey was disseminated online using Survey Monkey and Qualtrics software. All participants were anonymous. Participants were allowed to answer responses at their own pace (untimed) with the average of completion time of 30 minutes. All participants were afforded the option to retract at any time. Privacy and confidentiality for the online survey instruments was maintained by using a numbering system for coding surveys.

Data Coding

Mathematics teaching efficacy and outcome expectancy beliefs were measured using participant's responses to Likert-scale questions on the MTEBI. Each response was assigned a numerical value: 1 strongly disagree, 2 disagree, 3 neither agree nor disagree, 4 agree, and 5 for strongly agree. Items 3, 6, 8, 15, 17, 18, 19, and 21 of the MTEBI were reverse scored in order to produce consistent values between positively and negatively worded items (Enochs et al., 2000). Reverse coding these items will produce high scores for those high in efficacy and outcome expectancy beliefs as well as low scores for those low in efficacy and outcome expectancy beliefs. Items were recoded as follows (5=1), (4=2), (3=3), (2=4), and (1=5). Both teaching efficacy and outcome expectancy responses of 1 or 2 were considered low and scores of 4 or 5 were considered high. Knowledge of algebra for teaching was measured using participant's responses to multiple choice math questions on the 15 KAT items. Each response was assigned a numerical value: 0 incorrect and 1 correct.

Data Analysis Procedures

Initially data were analyzed for inclusion criteria (i.e. teachers who teach middle or high school mathematics will be included while those who do not teach middle or high school mathematics will be excluded). Preliminary analyses were conducted to observe the mean and standard deviations of all variables (teaching experience, teaching certification, teaching level, gender, ethnicity, age, etc.). Additionally, descriptive statistics (frequencies, percentages, means, and standard deviations) were used to analyze the quantitative data, and the findings were displayed through a series of tables. After conducting a preliminary data analysis, each of the three research questions in the study were analyzed. Results of the preliminary data analysis, description of the analyses used with each of the three research questions, final data analysis and results of these analyses can be found in Chapter IV.

Regression analyses. A multiple regression analysis was conducted to identify differences among responses from secondary math teachers, using individual teacher characteristics as the independent variables, and 1) mathematics teaching efficacy, 2) outcome expectancy, and 3) knowledge of algebra for teaching as the dependent variables respectively. A hierarchical multiple regression analysis was conducted to determine whether individual teacher characteristics and Knowledge of Algebra for Teaching (KAT) can predict teaching efficacy and outcome expectancy beliefs. Scores on the 1) teaching efficacy and 2) outcome expectancy subscale of the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) were used as the dependent variable and teacher characteristics and Knowledge of Algebra for Teaching (KAT) scores were entered as the independent variable in two separate block models.

Mediation analyses. Mediation effects were tested following the approach identified by Preacher & Hayes, (2004); Preacher & Hayes (2008); Hayes, (2013) in which estimation and interpretation of the direct, indirect, and total effects with a normal theory approach and a bootstrap approach to obtaining confidence intervals will occur. The process began by estimating the direct effect of teacher characteristics on teaching efficacy, the indirect effect of teaching characteristics on teaching efficacy transmitted through knowledge of algebra for teaching, followed by combining the direct and indirect effect of teaching characteristics on teaching efficacy which resulted in the total effect. Following these steps, a second procedure began by estimating the direct effect of teacher characteristics on outcome expectancy, the indirect effect of teaching characteristics on outcome expectancy transmitted through knowledge of algebra for teaching, followed by combining the direct and indirect effect of teaching characteristics on outcome expectancy which resulted in the total effect.

In conducting the mediation analyses, the bootstrap approach was used. This approach is particularly useful relative to the normal theory approach in smaller samples, because in smaller samples the non-normality of the sampling distribution of the indirect effect is likely to be most severe, the large sample asymptotics of the normal theory approach are harder to trust, and the power advantages of bootstrapping are more pronounced (Hayes, 2013).

The purpose of this study was to measure and compare the levels of teaching efficacy, outcome expectancy, and knowledge of algebra for teaching of secondary mathematics teachers. In addition, the relationship between teacher efficacy, outcome expectancy, knowledge of algebra for teaching and other teachers' demographic characteristics were studied. The research design, methodology, data collection, and data analysis procedures were summarized and the selection process for the research participants, and the validity and reliability of the study's instruments were provided in the present chapter.

Chapter IV

Results

The present study measured and compared the levels of teaching efficacy, outcome expectancy, and knowledge of algebra for teaching amongst pre-service, in-service, and former secondary mathematics teachers. Teacher's efficacy and outcome expectancy was measured using Enochs, Smith, & Huinker's (2000) Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). Algebra knowledge was measured using McCrory et al., (2012) Knowledge of Algebra for Teaching (KAT) instrument. Analyses using the Statistical Package for Social Sciences (SPSS Version 23) are offered in the present section. First, a description of the factor analyses involving the MTEBI and KAT scales are presented in Tables 2-5. Second, scale reliabilities, descriptive statistics, and bivariate correlations are presented in Tables 6-11. Next, results of hierarchical linear regressions of are presented in Tables 12-14 and Figures 5-9. Finally, results of mediation analyses of KAT on teacher characteristics and MTEBI subscales are presented in Tables 15-18 and Figures 10-16.

Principal Components Analysis

Mathematics teaching efficacy beliefs instrument. An exploratory factor analysis using the principal components method with varimax rotation was conducted with the participant's scores on the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) items. Initially, the factorability of the 21 MTEBI items was examined. Each of the 21 items correlated with a minimum of .3 with another item, suggesting reasonable factorability. The Kaiser-Meyer-Olkin (KMO) Measure of sampling adequacy was .66, above the recommended value of .6, and a Bartlett's Test of Sphericity was significant

($\chi^2(210)=540.884$, $p< .05$).

The primary purpose was to determine if variations in participants scores correspond with the proposed two factors (teaching efficacy and outcome expectancy) within the MTEBI items as hypothesized in the Enochs et al., (2000) theoretical framework. The initial eigenvalues showed that the first factor explained 18.7% of the variance and the second factor explained 14.2% of the variance. The two factor solution which explained 32.9% of the variance was preferred because of its previous theoretical support. Results of the analysis demonstrated that within the two identified factors, items 11 and 15 did not produce a factor loading greater than .3, indicating that these items may not measure teaching efficacy as previous research shows. Item 11 *"I understand mathematics concepts well enough to be effective in teaching mathematics"* did not produce a strong loading on either factor, and item 15 *"I find it difficult to use manipulatives to explain to students why mathematics works"* was cross loading on both factors (Table 2). Due to item 15 cross loading on both components in the principal components analysis, the item was not retained. Item 11 was retained because although a loading of .272 was not strong, it was much higher than the .115 loading that item 11 displayed on the second component. Additionally, a factor loading of .272 was near the .30 recommendation.

Table 2
Rotated Component Matrix for MTEBI Principal Components Analysis

Item	Comp 1	Comp 2
1. When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort - (OE)	-.075	**.432**
2. I am continually finding better ways to teach mathematics – (TE)	**.320**	.170
3. Even if I try very hard, I will not teach mathematics as well as I will most subjects – (TE)	**.543**	.151
4. When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach - (OE)	-.092	**.600**
5. I know how to teach mathematics concepts effectively – (TE)	**.530**	.082
6. I am not very effective in monitoring mathematics activities – (TE)	**.657**	.041
7. If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching - (OE)	.026	**.568**
8. I generally teach mathematics ineffectively – (TE)	**.540**	.218
9. The inadequacy of a student's mathematics background can be overcome by good teaching - (OE)	.242	**.586**
10. When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher - (OE)	-.002	**.558**
11. I understand mathematics concepts well enough to be effective in teaching mathematics – (TE)	**.272**	.115
12. The teacher is generally responsible for the achievement of students in mathematics - (OE)	.261	**.691**
13. Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching - (OE)	.094	**.646**
14. If parents comment that their child is showing more interest in mathematics in school, it is probably due to the performance of the child's teacher - (OE)	.132	**.434**
15. I find it difficult to use manipulatives to explain to students why mathematics works – (TE)	.297	**-.369**
16. I typically am able to answer students' questions about mathematics – (TE)	**.430**	-.105
17. I wonder if I have the necessary skills to teach mathematics – (TE)	**.717**	-.095
18. Given a choice, I would not invite the principle to evaluate my mathematics teaching – (TE)	**.696**	.043
19. When a student has difficulty understanding a mathematics concept, I usually am at a loss as to how to help the student understand better – (TE)	**.719**	-.197
20. When teaching mathematics, I usually welcome student questions – (TE)	**.572**	-.193
21. I do not know what to do to turn students on to mathematics – (TE)	**.415**	.173

Note. TE = Teaching Efficacy and OE = Outcome Expectancy

Knowledge of algebra for teaching. McCrory et al., (2012) propose three categories of knowledge essential to effective teaching of algebra: school knowledge of algebra, advanced knowledge of mathematics, and algebra knowledge for teaching. The three domains of KAT represent an extension of the domains identified by various researchers (school knowledge and teaching knowledge) for elementary teachers, to include the advanced mathematical knowledge (e.g., calculus and abstract algebra) recommended for algebra teachers in middle school and high school. An exploratory factor analysis using the principal components method with varimax rotation was conducted with the participants scores on the KAT items. Initially, the factorability of the 15 KAT items was examined. Each of the 15 items correlated with a minimum of .3 with another item, suggesting reasonable factorability. The Kaiser-Meyer-Olkin (KMO) Measure of sampling adequacy was .63, above the recommended value of .6, and a Bartlett's Test of Sphericity was significant ($\chi^2(105)=231.258$, $p< .05$).

Initially, eigenvalues greater than 1 were extracted as an indicator of how many factors to retain, in which 6 components were identified. In the six factor model, the initial eigenvalues showed that the first factor explained 20.3% of the variance, the second factor explained 10.6% of the variance, the third factor explained 9.0% of the variance, the fourth factor explained 8.0% of the variance, the fifth factor explained 7.3% of the variance, and the sixth factor explained 7.1% of the variance, with the total six-factor solution explaining 62.4% of the variance. However only one item loaded onto component 6 and only two items loaded onto components 2, 4, and 5 (Table 3). According to Costello & Osborne (2005), a factor with fewer than three items is generally weak and unstable, thus the six-factor solution was not retained.

Table 3
Rotated Component Matrix for KAT Principal Components Analysis Eigenvalues > 1

Item	Component 1	2	3	4	5	6
1. Express quantitative relationships in word problems using algebraic expressions – (SK)	.021	-.035	.058	-.007	-.024	**.865**
2. Solve quadratic equations $2x^2 = 6x$ (losing roots) – (TK)	**.599**	.254	.038	.159	.248	.283
3. Given a quadratic function f(x), find f(x + a) – (SK)	.310	-.070	.057	**.717**	.258	.241
4. Transform $f(x) = \log_2 x^2$ - (AK)	-.100	**.573**	.296	.123	.043	-.195
5. Solve equation: $9^x - 3^x - 6 = 0$ using substitution method (adding roots) – (TK)	.380	-.021	**.665**	-.383	-.124	-.055
6. Represent fractions, percents, and algebraic expressions such as 3/5, 60%, and a(b + c) = ab + ac using the area of rectangle – (SK)	.007	.386	**.588**	.121	-.001	.363
7. Given two points, find the functions whose graphs passing these two points – (TK)	**.572**	-.008	.241	.196	.368	.076
8. Given a graph representing speed vs. time for two cars, judge the position of the two cars – (AK)	.030	**.797**	-.031	.034	.121	.146
9. Judge the number of root of equation: $\tan x = x^2$ – (AK)	-.097	.000	**.735**	.211	.109	-.009
10. Judge perpendicular relationship of two lines by using their slopes- (TK)	**.733**	.051	.013	.112	.069	-.159
11. Multiple ways to introduce the concept of slope a line – (TK)	.185	.212	.137	**.731**	-.240	-.188
12. Judge the proposition "For all a and b in S, if ab=0, then either a=0 or b=0" in different number systems – (AK)	.032	.181	.011	.013	**.797**	-.007
13. Meaning of mathematical induction	.320	**.541**	-.013	-.088	.397	-.164
14. Roots of irrational equation $\sqrt{x} - 2 = \sqrt{1} - x$ (adding roots) – (SK)	**.424**	.401	-.119	.087	-.329	.327
15. Expand algebra expressions by area relationship – (TK)	**.548**	-.094	-.081	.066	-.280	.070

Note. SK = School Knowledge, AK = Advanced Knowledge, and TK = Teaching Knowledge

Next, a fixed number of three factors were extracted. The purpose of extracting three factors was to determine if variations in participants KAT scores corresponded with the proposed three tenets (school knowledge, advanced knowledge, and teaching knowledge) within the knowledge of algebra for teaching construct as hypothesized in the (McCrory et al., 2012), theoretical framework. Justification of the three proposed tenets were made at the item writers (Reckase, McCrory, Floden, Ferrini-Mundy, & Senk, 2015) discretion. In the three-factor model, all items produced a minimum of .3 factor loading and the initial eigenvalues showed that the first factor explained 20.3% of the variance, the second factor explained 10.6% of the variance, and the third factor explained 9.0% of the variance, with the total three factor solution explaining 39.9% of the variance. Upon reviewing these items, it was unclear whether components assessed school knowledge, advanced knowledge, or teaching knowledge thus limiting rationale for retaining a three-factor solution.

Table 4

Rotated Component Matrix for KAT Principal Components Analysis Three Fixed Factors

Item	Component 1	Component 2	Component 3
1. Express quantitative relationships in word problems using algebraic expressions – (SK)	.222	-.261	**.351**
2. Solve quadratic equations $2x^2 = 6x$ (losing roots) – (TK)	**.680**	.269	.100
3. Given a quadratic function f(x), find f(x + a) – (SK)	**.615**	.066	.108
4. Transform $f(x) = \log_2 x^2$ - (AK)	-.057	**.507**	.328
5. Solve equation: $9^x - 3^x - 6 = 0$ using substitution method (adding roots) – (TK)	.125	-.055	**.514**
6. Represent fractions, percents, and algebraic expressions such as 3/5, 60%, and a(b + c) = ab + ac using the area of rectangle – (SK)	.142	.214	**.741**
7. Given two points, find the functions whose graphs passing these two points – (TK)	**.591**	.223	.136
8. Given a graph representing speed vs. time for two cars, judge the position of the two cars – (AK)	.141	**.612**	.175
9. Judge the number of root of equation: $\tan x = x^2$ – (AK)	-.041	.114	**.657**
10. Judge perpendicular relationship of two lines by using their slopes- (TK)	**.653**	.119	-.109
11. Multiple ways to introduce the concept of slope a line – (TK)	**.415**	.063	.186
12. Judge the proposition "For all a and b in S, if ab=0, then either a=0 or b=0" in different number systems – (AK)	.058	**.646**	-.091
13. Meaning of mathematical induction	.254	**.682**	-.061
14. Roots of irrational equation $\sqrt{x - 2} = \sqrt{1 - x}$ (adding roots) – (SK)	**.519**	-.016	.117
15. Expand algebra expressions by area relationship – (TK)	**.516**	-.273	-.069

Note. SK = School Knowledge, AK = Advanced Knowledge, and TK = Teaching Knowledge

Based on the initial extraction of eigenvalues greater than one in which six components were identified, components with less than three items (components 2, 4, 5, and 6) were eliminated and a fixed number of two factors were extracted. In the two factor model, the initial eigenvalues showed that the first factor explained 20.3% of the variance, the second factor explained 10.6% of the variance, with the total two factor solution explaining 30.9% of the variance. However, in the two factor model item 1 and item 5 did not produce a factor loading above .3 onto any component. Ultimately a one factor solution was deemed adequate based on higher reliability as presented in the following section.

Table 5
Rotated Component Matrix for KAT Principal Components Analysis Two Fixed Factors

Item	Component 1	Component 2
1. Express quantitative relationships in word problems using algebraic expressions – (SK)	**.264**	-.036
2. Solve quadratic equations $2x^2 = 6x$ (losing roots) – (TK)	**.664**	.301
3. Given a quadratic function f(x), find f(x + a) – (SK)	**.615**	.129
4. Transform $f(x) = \log_2 x^2$ - (AK)	-.072	**.602**
5. Solve equation: $9^x - 3^x - 6 = 0$ using substitution method (adding roots) – (TK)	.163	**.221**
6. Represent fractions, percents, and algebraic expressions such as 3/5, 60%, and a(b + c) = ab + ac using the area of rectangle – (SK)	.176	**.570**
7. Given two points, find the functions whose graphs passing these two points – (TK)	**.581**	.277
8. Given a graph representing speed vs. time for two cars, judge the position of the two cars – (AK)	.108	**.617**
9. Judge the number of root of equation: $\tan x = x^2$ – (AK)	-.004	**.435**
10. Judge perpendicular relationship of two lines by using their slopes- (TK)	**.634**	.063
11. Multiple ways to introduce the concept of slope a line – (TK)	**.421**	.161
12. Judge the proposition "For all a and b in S, if ab=0, then either a=0 or b=0" in different number systems – (AK)	.004	**.507**
13. Meaning of mathematical induction	.199	**.559**
14. Roots of irrational equation $\sqrt{x - 2} = \sqrt{1 - x}$ (adding roots) – (SK)	**.526**	.061
15. Expand algebra expressions by area relationship – (TK)	**.529**	-.255

Note. SK = School Knowledge, AK = Advanced Knowledge, and TK = Teaching Knowledge

Reliability Analysis

Mathematics teaching efficacy beliefs instrument. An initial reliability analysis was performed on all 21 MTEBI items. Of the items, thirteen are considered efficacy items (questions 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, and 21) and eight are considered outcome expectancy items (questions 1, 4, 7, 9, 10, 12, 13, and 14). The reliability of all 21 items within the instrument was .751. Item 15, *"I find it difficult to use manipulatives to explain to students why mathematics works"* produced low item-total correlations (r=.036) and the Cronbach's alpha if the item was deleted would increase to .764. A secondary reliability analysis was conducted based on the two factors extracted in the researcher's principal components analysis and the theoretical framework suggested by Enochs et al., (2000). The reliability for the teaching efficacy subscale was .774 (with item 15 deleted) and the reliability for the outcome expectancy subscale was .724.

Knowledge of algebra for teaching. Based on theoretical support, a reliability analysis was performed using the three proposed tenets of KAT suggested by (McCrory et al., 2012). School knowledge items (questions 1, 3, 6, and 14), advanced knowledge items (questions 4, 8, 9, 12, and 13) and teaching knowledge items (questions 2, 5, 7, 10, 11, and 15) produced a reliability of .421, .574, and .510 respectively. Given the low reliabilities on the three subscales, an additional reliability analysis was conducted using KAT as a single component in which reliability of all 15 items within the instrument was acceptable with a value of .688.

Table 6
Cronbach Alphas for Scales

Scale	Number of Items	Cronbach Alpha
Mathematics Teaching Efficacy Beliefs Instrument	20	.76
Teaching Efficacy	12	.77
Outcome Expectancy	8	.72
Knowledge of Algebra for Teaching	15	.69
School Knowledge	4	.42
Advanced Knowledge	5	.57
Teaching Knowledge	6	.51

Descriptive Statistics

To obtain descriptive statistics on the sample, analyses were conducted to determine the normality of distribution of scores on each of the variables for which data were collected. The mean scores were above the midpoint for all of the MTEBI items. Tables 7 and 8 show that participants demonstrated high teaching efficacy and somewhat neutral outcome expectancy as measured by MTEBI. Each response was assigned a numerical value: 1 strongly disagree, 2 disagree, 3 neither agree nor disagree, 4 agree, and 5 for strongly agree. Both teaching efficacy and outcome expectancy responses of 1 or 2 were considered low and scores of 4 or 5 were considered high. Items 3, 6, 8, 15, 17, 18, 19, and 21 of the MTEBI were reverse scored in order to produce consistent values between positively and negatively worded items (Enochs et al., 2000).

Table 7
Means and Standard Deviation (Teaching Efficacy)

Item	Mean	SD
2. I am continually finding better ways to teach mathematics	4.52	.54
3. Even if I try very hard, I will not teach mathematics as well as I will most subjects	4.69	.63
5 I know how to teach mathematics concepts effectively	4.25	.52
6 I am not very effective in monitoring mathematics activities	4.22	.66
8 I generally teach mathematics ineffectively	4.46	.72
11 I understand mathematics concepts well enough to be effective in teaching mathematics	4.22	.88
15 I find it difficult to use manipulatives to explain to students why mathematics works	4.61	.60
16 I typically am able to answer students' questions about mathematics	4.24	.98
17 I wonder if I have the necessary skills to teach mathematics	4.29	.97
18 Given a choice, I would not invite the principle to evaluate my mathematics teaching	4.37	.73
19 When a student has difficulty understanding a mathematics concept, I usually am at a loss as to how to help the student understand better	4.72	.45
20 When teaching mathematics, I usually welcome student questions	3.79	.92

Table 8
Means and Standard Deviation (Outcome Expectancy)

Item	Mean	SD
1 When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort	3.42	.88
4 When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach	3.79	.62
7 If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching	2.88	.98
9 The inadequacy of a student's mathematics background can be overcome by good teaching	3.82	.93
10 When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher	3.74	.79
12 The teacher is generally responsible for the achievement of students in mathematics	3.41	.92
13 Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching	3.45	.96
14 If parents comment that their child is showing more interest in mathematics in school, it is probably due to the performance of the child's teacher	3.72	.79

The means and standard deviations displayed in Table 9 show that participants performed best on KAT items 1, 2, 3, 7, 10, and 11 scoring above 60% accuracy and performed poorly on items 4, 5, 8, 13, and 15 scoring below 40% accuracy. Each response was assigned a numerical value of 1 for a correct response and 0 for an incorrect response. Upon closer examination it is evident that 60% of the advanced knowledge items had no more than a 46% accuracy amongst participants.

Table 9
Means and Standard Deviations (KAT)

Item	Mean	SD
1. Express quantitative relationships in word problems using algebraic expressions – (SK)	.82	.39
2. Solve quadratic equations $2x^2 = 6x$ (losing roots) – (TK)	.69	.47
3. Given a quadratic function f(x), find f(x + a) – (SK)	.87	.34
4. Transform $f(x) = \log_2 x^2$ - (AK)	.32	.47
5. Solve equation: $9^x - 3^x - 6 = 0$ using substitution method (adding roots) – (TK)	.35	.48
6. Represent fractions, percents, and algebraic expressions such as 3/5, 60%, and a(b + c) = ab + ac using the area of rectangle – (SK)	.55	.50
7. Given two points, find the functions whose graphs passing these two points – (TK)	.65	.48
8. Given a graph representing speed vs. time for two cars, judge the position of the two cars – (AK)	.31	.47
9. Judge the number of root of equation: $\tan x = x^2$ – (AK)	.46	.50
10. Judge perpendicular relationship of two lines by using their slopes- (TK)	.77	.42
11. Multiple ways to introduce the concept of slope a line – (TK)	.76	.43
12. Judge the proposition "For all a and b in S, if ab=0, then either a=0 or b=0" in different number systems – (AK)	.42	.50
13. Meaning of mathematical induction	.38	.49
14. Roots of irrational equation $\sqrt{x-2} = \sqrt{1-x}$ (adding roots) – (SK)	.51	.50
15. Expand algebra expressions by area relationship – (TK)	.32	.47

A correlation matrix using Pearson's product moment correlation was constructed and examined to determine how teaching characteristics, teaching efficacy, outcome expectancy, and knowledge of algebra for teaching variables were correlated. A significant negative relationship was found among participant's responses on the knowledge of algebra for teaching instrument and their outcome expectancy scores (Table 10). Years of experience, math teaching certification, years of education, teacher type, and age showed significant relationships with teaching efficacy scores, public school teacher's showed a significant relationship with outcome expectancy scores, and knowledge of algebra for teaching scores showed a significant relationship among high school teachers and certified math teachers (Table 11).

Table 10
Correlations Among MTEBI and KAT (N=100)

	1	2	3
1. Teaching Efficacy	-	.177	.141
2. Outcome Expectancy		-	-.197*
3. KAT			-

Note. **p* < .05. ***p* < .01

Table 11

Correlations Between Teaching Efficacy, Outcome Expectancy, KAT, and Teacher Characteristics Variables

	HS Teacher	Years of Experience	Certified Math	Years of Education	Public School Teacher	Ethnicity (White)	Teacher Type	Age	Gender
Teaching Efficacy	-.069	.264**	.235*	.403**	.069	.086	.213*	.320**	.099
Outcome Expectancy	-.118	.020	-.116	.092	-.221*	-.128	.038	.093	.140
KAT	.468**	.121	.273**	.177	-.179	.105	.083	.101	-.130

Note. *$p < .05$. **$p < .01$

Hierarchical Linear Regressions

To determine whether there were differences among secondary math teacher's teaching characteristics such as grade level taught, gender, ethnicity, years of experience, and certification, in terms of knowledge of algebra for teaching, teaching efficacy, and outcome expectancy the researcher performed multiple regression analyses. Additionally, to determine whether teacher characteristics and knowledge of algebra for teaching significantly predicted teaching efficacy and outcome expectancy beliefs the researcher performed hierarchical regression analyses.

Teaching efficacy. An initial linear regression addressed teaching characteristics and teaching efficacy. A significant regression equation was found among efficacy subscale scores and having a math teaching certification, and efficacy subscale scores and years of education, ($F(9, 90)=3.362$ $p<.05$) with an R^2 of .252. Upon including Knowledge of Algebra for Teaching (KAT) scores into the model, a significant regression equation was found among teaching efficacy subscale scores and years of education $F(1, 89)=3.144$, $p<.05$) with an R^2 of .261. Thus, math teaching certification and years of education were significant predictors of teaching efficacy however with the addition of KAT, math teaching certification lost it's significance.

Table 12
Hierarchical Regression Analyses for Variables Predicting Teaching Efficacy

	Model 1			Model 2		
	B	*SE B*	*β*	*B*	*SE B*	*β*
High School Teacher	-.092	.080	-.115	-.137	.091	-.172
Years of Experience	-.005	.008	-.094	-.005	.008	-.091
Certified in Math	.238	.116	.255*	.225	.116	.241
Years of Education	.074	.030	.299*	.069	.030	.277*
Public School Teacher	.084	.106	.074	.109	.108	.096
Ethnicity (White)	.097	.089	.106	.076	.091	.083
Teacher Type	-.135	.113	-.141	-.137	.113	-.144
Age	.010	.006	.298	.010	.006	.300
Gender	.086	.095	.088	.087	.095	.089
KAT				.238	.223	.120
R^2		.252			.261	
F		3.362			3.144	

Note. $*p < .05$. $**p < .01$

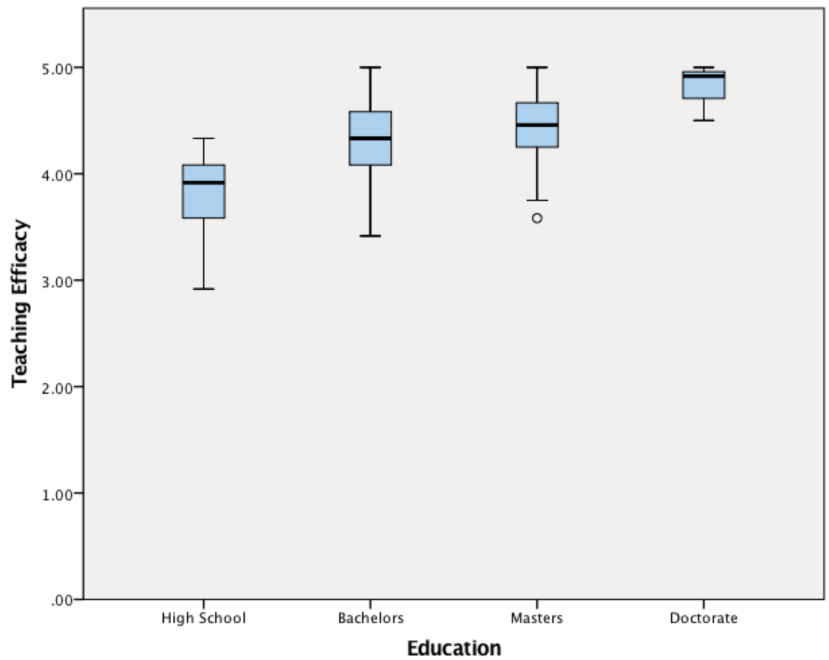


Figure 4. Years of Education and Teaching Efficacy Box Plot

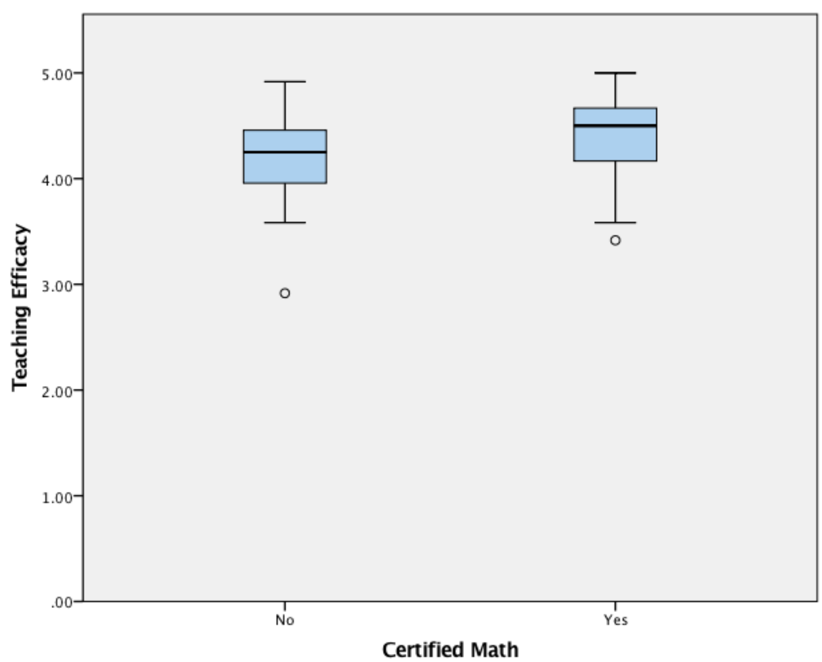


Figure 5. Math Certification and Teaching Efficacy Box Plot

Outcome expectancy. An initial linear regression addressed teaching characteristics and outcome efficacy. A significant regression equation was found among expectancy subscale scores and being a public school teacher, ($F(9, 90)=1.334$, $p<.05$) with an R^2 of .118. Upon including Knowledge of Algebra for Teaching (KAT) scores into the model, a significant regression equation was also found among outcome expectancy subscale scores, being a public school teacher, and KAT $F(10, 89)=1.639$, $p<.05$) with an R^2 of .156. Thus, being a public school teacher was a significant predictor of outcome expectancy.

Table 13
Hierarchical Regression Analyses for Variables Predicting Outcome Expectancy

	Model 1			Model 2		
	B	*SE B*	*β*	*B*	*SE B*	*β*
High School Teacher	-.090	.111	-.088	.026	.124	.026
Years of Experience	-.003	.012	-.045	-.003	.011	-.051
Certified in Math	-.120	.161	-.100	-.086	.159	-.072
Years of Education	.003	.041	.011	.017	.041	.053
Public School Teacher	-.332	.146	-.229*	-.397	.148	-.274*
Ethnicity (White)	-.150	.124	-.129	-.097	.125	-.084
Teacher Type	.094	.157	.077	.101	.155	.082
Age	.005	.008	.125	.005	.008	.121
Gender	.125	.132	.099	.123	.130	.098
KAT				-.607	.304	-.239*
R^2		.118			.156	
F		1.334			1.639	

Note. $*p < .05$. $**p < .01$

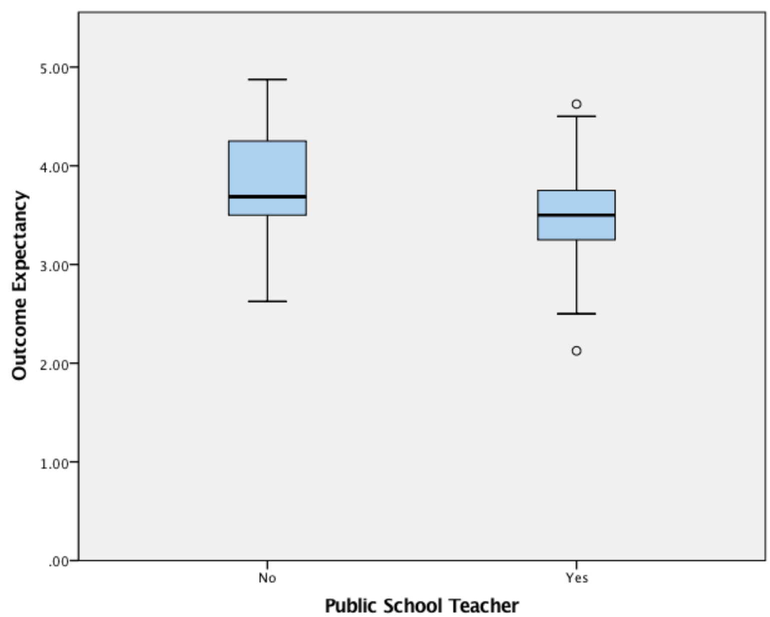


Figure 6. Public School Teacher and Outcome Expectancy Box Plot

Knowledge of algebra for teaching. A significant regression equation was found among KAT scores and identifying as a high school teacher, (F(9, 90)=5.134, p<.001) with an R^2 of .339. A significant regression equation was also found among KAT scores and identifying as a public school teacher, and a white teacher (F(9, 90)= 5.134, p<.05) with an R^2 of .339. Thus, being a high school teacher, public school teacher, and a white teacher were significant predictors of knowledge of algebra for teaching.

Table 14
Regression Analyses for Variables Predicting KAT Scores

	B	*SE B*	*β*
High School Teacher	.192	.038	.478***
Years of Experience	-.001	.004	-.024
Certified in Math	.056	.055	.120
Years of Education	.022	.014	.178
Public School Teacher	-.108	.050	-.188*
Ethnicity (White)	.087	.042	.190*
Teacher Type	.010	.054	.021
Age	.000	.003	-.018
Gender	-.003	.045	-.006
R^2		.339	
F		5.134	

Note. $*p < .05$. $**p < .01$. $***p < .001$.

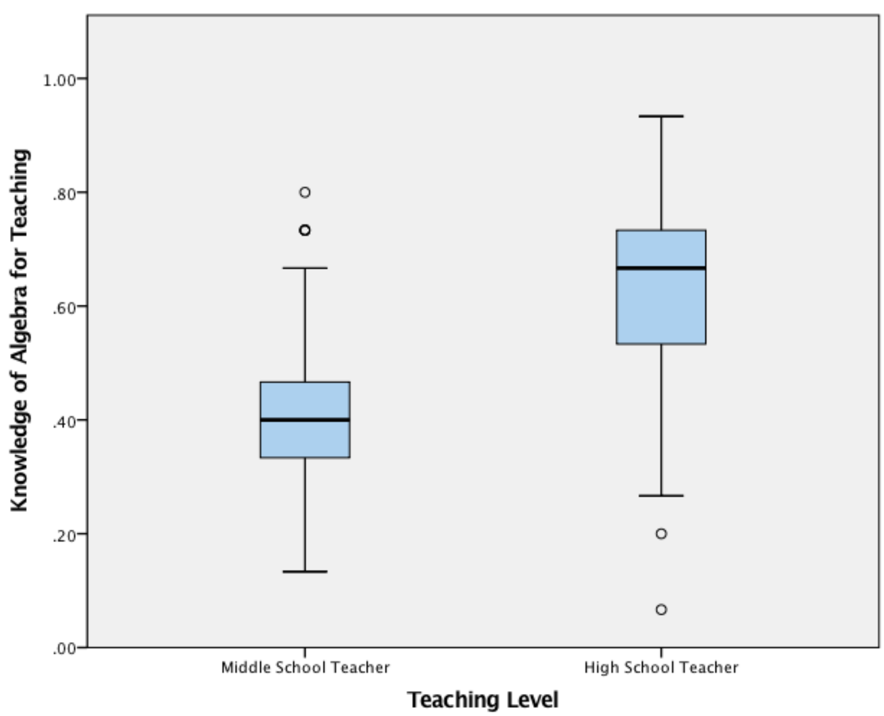


Figure 7. Teaching Level and Knowledge of Algebra for Teaching Box Plot

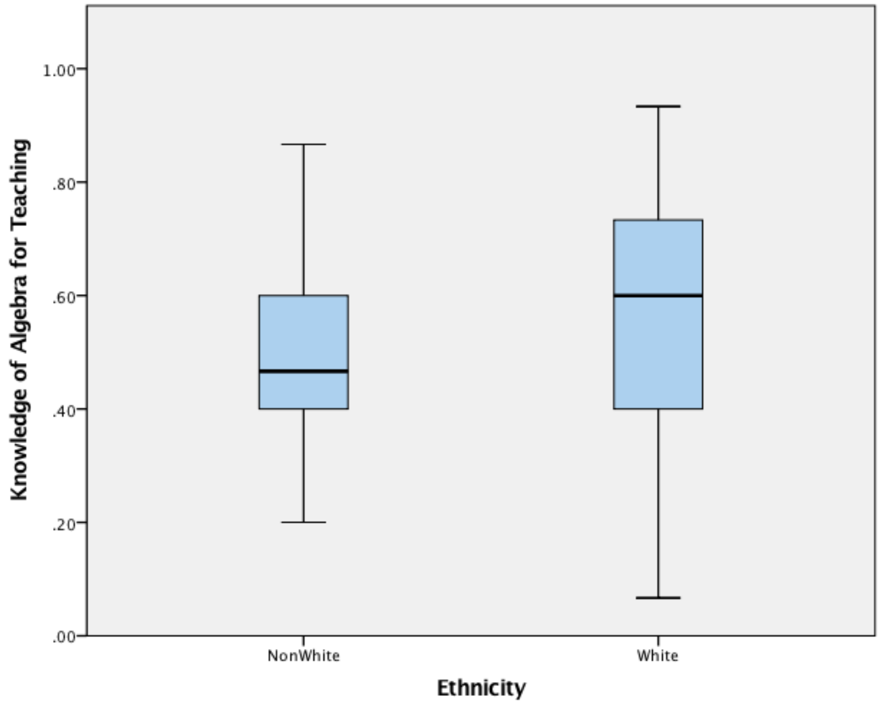


Figure 8. Ethnicity and Knowledge of Algebra for Teaching Box Plot

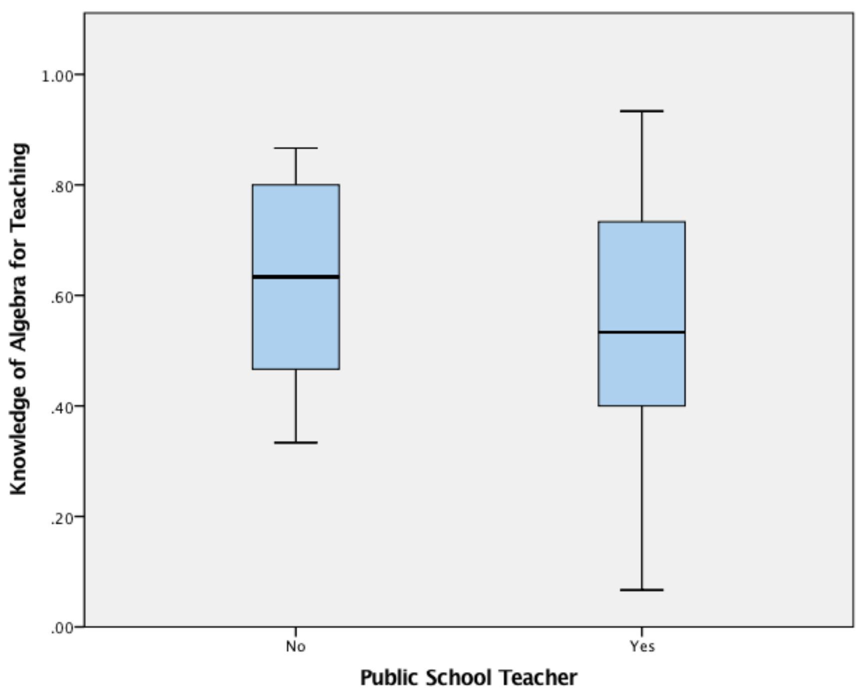


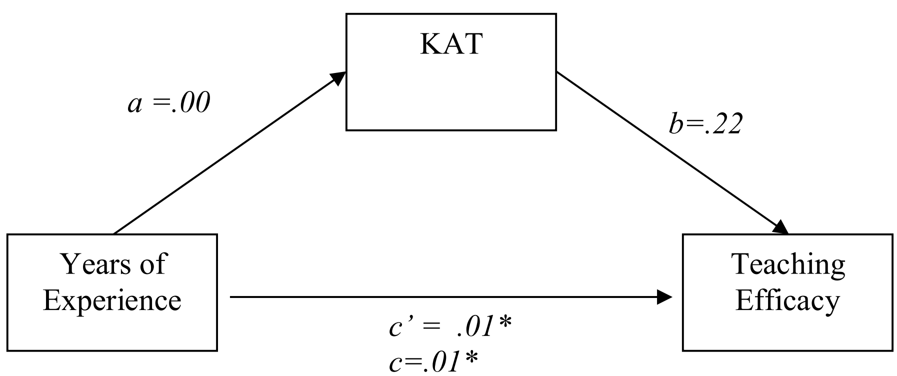
Figure 9. Public School Teacher and Knowledge of Algebra for Teaching Box Plot

Mediation Analyses

To determine whether knowledge of algebra for teaching produced an indirect effect on the relation between teacher characteristics and MTEBI subscales (teaching efficacy and outcome expectancy) the researcher performed mediation analyses.

Teaching efficacy and teacher characteristics bivariate correlations and regression. An initial correlation report was analyzed in order to identify which teacher characteristics related to teaching efficacy. Followed by a linear regression analysis to predict individual teacher characteristics and teaching efficacy. Results of the correlation report and regression analysis are found in Tables 11 and 12. The correlation report determined that years of teaching experience, teachers certified in math, years of education, teacher type, and age were significantly correlated with teaching efficacy. The regression analysis determined that math certification and years of education were significant predictors of teaching efficacy. These five variables were used in separate mediation models to determine whether or not knowledge of algebra for teaching mediated their relationship. The researcher chose not to put multiple independent variables in the mediation model based on Hayes (2013) assertion that including multiple independent variables in a mediation model, increases the possibility that highly correlated independent variables will cancel out each other's effects.

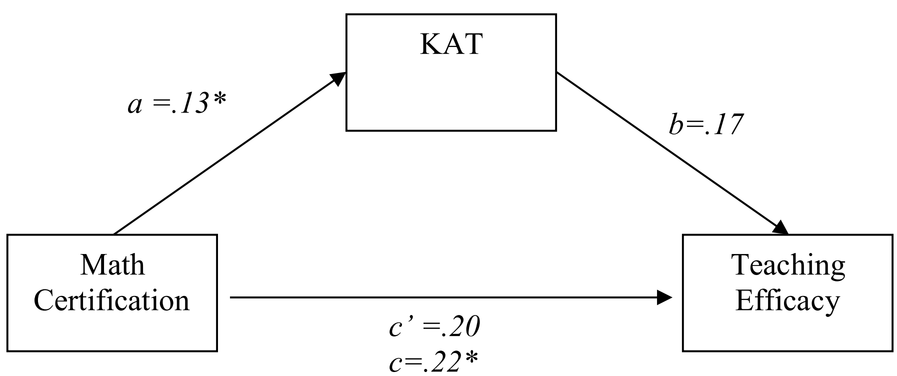
Years of experience. The standardized regression coefficient (path a) between years of teaching experience and KAT was not statistically significant. The standardized regression coefficient between KAT and teaching efficacy (path b) was not significant. There was not a significant indirect effect of years of experience on teaching efficacy through KAT, *ab* = 0.0007, a bias-corrected bootstrap confidence interval for the indirect effect based on 1,000 bootstrap samples was below zero (-0.004 to 0.0037). However, there was evidence of a significant direct effect showing that teacher's years of teaching experience influenced teaching efficacy independent of its effect on KAT (*c'* = 0.0130, *p* = .03). The mediator accounted for less than one-tenth of the total effect M=0.05



Note. *p* < .05*, p < .01**, p < .001***

Figure 10. Mediation Model for Teaching Efficacy, Years of Experience, and KAT

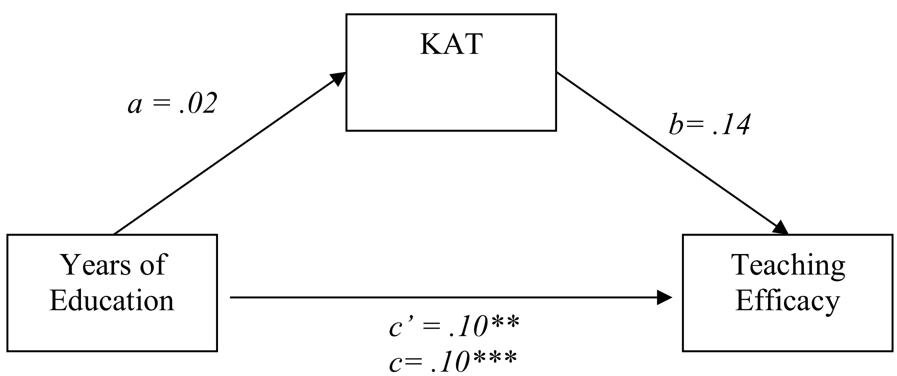
Math certification. The standardized regression coefficient between math certification and KAT was statistically significant, however the standardized regression coefficient between KAT and teaching efficacy was not significant (Table 16). There was not a significant indirect effect of math certification on teaching efficacy through KAT, *ab* = 0.0212, a bias-corrected bootstrap confidence interval for the indirect effect based on 1,000 bootstrap samples was below zero (-0.0210 to 0.0903). However, there was no evidence of a significant direct effect showing that math certification influenced teaching efficacy independent of its effect on KAT (*c'* = 0.199, *p* = .069). The mediator accounted for one-tenth of the total effect M=0.10



Note. $p < .05$*, $p < .01$**, $p < .001$***

Figure 11. Mediation Model for Teaching Efficacy, Math Certification, and KAT

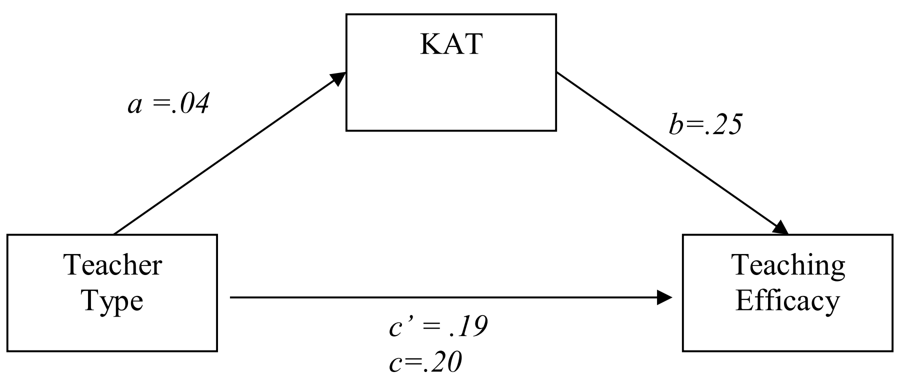
Years of education. The standardized regression coefficient between years of education and KAT was not statistically significant, the standardized regression coefficient between KAT and teaching efficacy was also not significant (Table 16). There was not a significant indirect effect of years of education on teaching efficacy through KAT, *ab* = 0.0032, a bias-corrected bootstrap confidence interval for the indirect effect based on 1,000 bootstrap samples was below zero (-0.0023 to 0.156). However, there was evidence of a significant direct effect showing that years of education influenced teaching efficacy independent of its effect on KAT (c' = 0.0971, p = .001). The mediator accounted for a small portion of the total effect M=0.03



Note. $p < .05$*, p < .01**, p < .001***

Figure 12. Mediation Model for Teaching Efficacy, Years of Education, and KAT

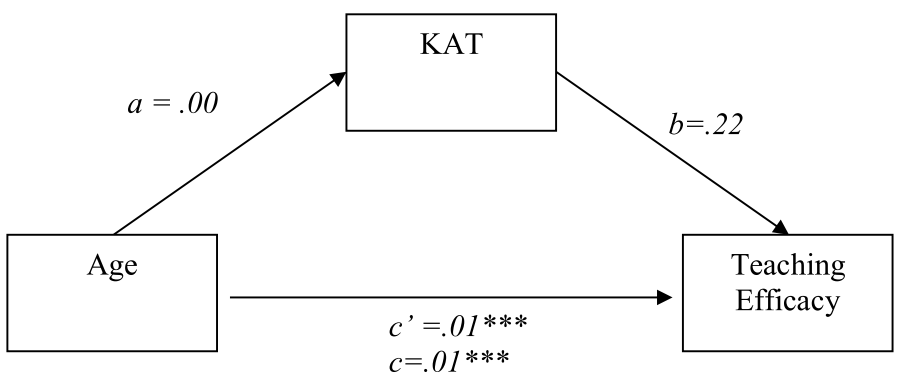
Teacher type. The standardized regression coefficient between teacher type and KAT was not statistically significant, the standardized regression coefficient between KAT and teaching efficacy was also not significant (Table 16). There was not a significant indirect effect of teacher type on teaching efficacy through KAT, *ab* = 0.0099, a bias-corrected bootstrap confidence interval for the indirect effect based on 1,000 bootstrap samples was below zero (-0.0102 to 0.0590). There was no evidence of a significant direct effect showing that teacher type influenced teaching efficacy independent of its effect on KAT (c' = 0.2030, p = .139). The mediator accounted for a small portion of the total effect M=0.06



Note. $p < .05$*, $p < .01$**, $p < .001$***

Figure 13. Mediation Model for Teaching Efficacy, Teacher Type, and KAT

Age. The standardized regression coefficient between age and KAT was not statistically significant, the standardized regression coefficient between KAT and teaching efficacy was also not significant (Table 16). There was not a significant indirect effect of age on teaching efficacy through KAT, *ab* = 0.004, a bias-corrected bootstrap confidence interval for the indirect effect based on 1,000 bootstrap samples was below zero (-0.0004 to 0.0024). However, there was evidence of a significant direct effect showing that age influenced teaching efficacy independent of its effect on KAT (*c'* = 0.0109, *p* = .0006). The mediator accounted for a small portion of the total effect M=0.03



Note. $p < .05$*, p < .01**, p < .001***

Figure 14. Mediation Model for Teaching Efficacy, Age, and KAT

Table 15
Regression coefficients, standard errors, and model summary for teaching efficacy mediation models

	Consequent							
	M(Knowledge of Algebra for Teaching)				*Y*(Teaching Efficacy)			
Antecedent		Coeff.	*SE*	*p*		Coeff.	*SE*	*p*
X_1(Years of Experience)	a_1	.003	.003	.236	$c_1{'}$	.014	.006	.020
M (KAT)					b_1	.220	.172	.202
Constant		.518	.032	.000		4.13	.111	.000
X_2(Math Certification)	a_2	.129	.050	.011	$c_2{'}$	.199	.109	.069
M (KAT)					b_2	.165	.183	.368
Constant		.446	.045	.000		4.12	.124	.000
X_3 (Years of Education)	a_3	.022	.013	.087	$c_3{'}$	.097	.029	.001
M (KAT)					b_3	.143	.164	.383
Constant		.172	.217	.428		2.65	.487	.000
X_4 (Teacher Type)	a_4	.040	.054	.462	$c_4{'}$	.193	.135	.155
M (KAT)					b_4	.247	.171	.151
Constant		.466	.111	.000		3.85	.300	.000
X_5 (Age)	a_5	.002	.002	.338	$c_5{'}$	.011	.003	.001
M (KAT)					b_5	.219	.160	.174
Constant		.483	.068	.000		3.864	.159	.000

Note. $p < .05$*, $p < .01$**, $p < .001$***

Table 16
Indirect effects, bootstrap standard errors, bootstrap lower and upper confidence intervals for teaching efficacy mediation models

	Y(Teaching Efficacy)			
	M(Knowledge for Algebra for Teaching)			
Antecedent	*Effect*	*Boot SE*	*Boot LLCI*	*Boot ULCI*
X_1(Years of Experience)	.0007	.0009	-.0004	.0037
X_2(Math Certification)	.0212	.0255	-.0210	.0903
X_3 (Years of Education)	.0032	.0043	-.0023	.0156
X_4 (Teacher Type)	.0099	.0159	-.0102	.0590
X_5 (Age)	.0004	.0005	-.0003	.0022

Outcome expectancy and teacher characteristics bivariate correlations and regression. An initial correlation report was analyzed in order to identify which teacher characteristics related to outcome expectancy. Followed by a linear regression analysis to predict individual teacher characteristics and outcome expectancy. Results of the correlation report and regression analysis are found in Tables 11 and 13. The correlation report determined that public school teachers were significantly negatively correlated with outcome expectancy, whereas participants KAT scores increased, their outcome expectancy decreased (Figure 15). The regression analysis determined that public school teachers were also significant negative predictors of outcome expectancy. The public teacher variable was used in a mediation models to determine whether or not knowledge of algebra for teaching mediated the relationship between identifying as public school teacher and outcome expectancy.

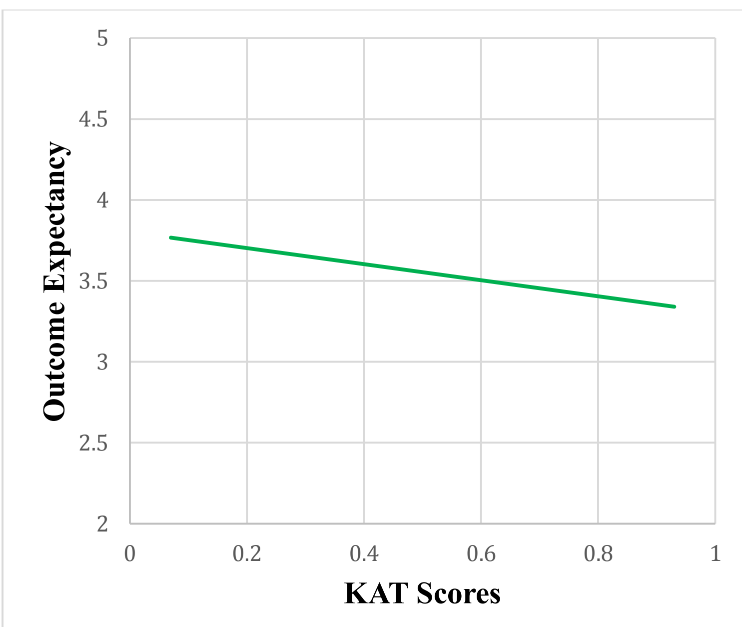
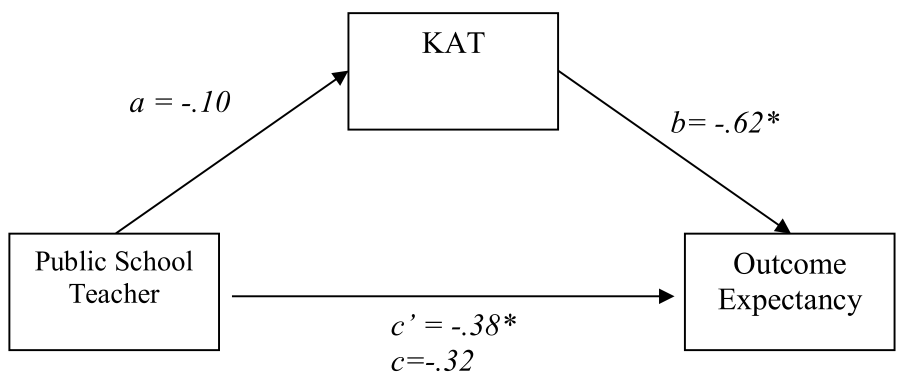


Figure 15. Regression Variable Plots KAT and Outcome Expectancy

Public school teacher. The relationship between public school teacher and outcome expectancy was mediated by KAT. The standardized regression coefficient between public school teacher and KAT was not statistically significant, however, the standardized regression coefficient between KAT and outcome expectancy was statistically significant (Table 20). There was a significant indirect effect of public school teacher on outcome expectancy through KAT, *ab* = 0.063, a bias-corrected bootstrap confidence interval for the indirect effect based on 1,000 bootstrap samples was above zero (0.004 to 0.179). There was also evidence of a significant direct effect showing that public school teacher influenced outcome expectancy independent of its effect on KAT (*c'* = -0.383, *p* = .034). The mediator accounted for roughly 20% of the total effect M=-0.20



Note. $p < .05$*, $p < .01$**, $p < .001$***

Figure 16. Mediation Model for Outcome Expectancy, Public School Teacher, and KAT

Table 17
Regression coefficients, standard errors, and model summary for outcome expectancy mediation models

	Consequent							
	M(Knowledge of Algebra for Teaching)				Y(Outcome Expectancy)			
Antecedent		Coeff.	*SE*	*p*		Coeff.	*SE*	*p*
X_1(Public School Teacher)	a_1	-.102	.054	.062	c_1'	-.383	.178	.034
M (KAT)					b_1	-.620	.256	.017
Constant		.633	.050	.000		4.196	.237	.000

Note. $p < .05$*, $p < .01$**, $p < .001$***

Table 18
Indirect effects, bootstrap standard errors, bootstrap lower and upper confidence intervals for outcome expectancy mediation models

	Y(Outcome Expectancy)			
	M(Knowledge for Algebra for Teaching)			
Antecedent	*Effect*	*Boot SE*	*Boot LLCI*	*Boot ULCI*
X_1(Public School Teacher)	.0634	.0430	.0044	.1792

Conclusion

In summary, having a math teaching certification and increased years of education leads to higher teaching efficacy. Public school teachers tend to have lower outcome expectancy scores. An important finding however was that bivariate correlations provided evidence that a statistically significant negative relationship exists between outcome expectancy and KAT ($r = -.197$, $p < .05$). There was no correlation between teaching efficacy and KAT. Additionally, the outcome expectancy scale was the only significant predictor of teacher's KAT scores. Chapter V includes further discussion of these results, limitations of the study, and recommendations for future studies.

Chapter V

Discussion

The present chapter will provide an overview of the study, followed by study findings and interpretations, limitations of the study, recommendations for future research, and discussion of implications.

Overview of the Study

The purpose of this study was to investigate the relationship between teaching efficacy, outcome expectancy, and knowledge of algebra for teaching among secondary mathematics teachers based on three proposed research questions. As indicated in the previous chapters, three types of data have been produced through the data collection process: (a) participant's responses to a demographic questionnaire, (b) participant's responses to the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) questionnaire, and (c) participant's responses to the Knowledge of Algebra for Teaching (KAT) questionnaire. The demographic questionnaire allowed for participants to respond to multiple choice, multiple response, and free response questions. The MTEBI questionnaire utilized a Likert-scale survey in which a score of 1=Strongly Disagree; 2=Disagree; 3=Uncertain; 4=Agree; 5=Strongly Agree. The KAT questionnaire utilized multiple choice algebra questions in which items were scored as either correct or incorrect with a maximum possible cumulative score of 100%. When evaluating MTEBI scores the researcher analyzed results based on the two components of the instrument, teaching efficacy and outcome expectancy. Additionally, when evaluating KAT scores the researcher analyzed results of the questionnaire in two ways, as a single component and as three proposed components – school knowledge, advanced knowledge, and teaching knowledge, based on theoretical background. All responses were analyzed using quantitative methods. Presently, an existing relationship between a teacher's knowledge of algebra for teaching, mathematics teaching efficacy, and outcome expectancy is unclear. The intent of the study was to contribute to the literature regarding these relationships specifically among secondary mathematics teachers.

Study Findings and Interpretations

Principal components analysis inferences. Results from factor analyses indicated that it is possible to separate the MTEBI scale into two components, teaching efficacy and outcome expectancy. Both the teaching efficacy scale and outcome expectancy scale produced high reliabilities. However, item 15, *"I find it difficult to use manipulatives to explain to students why mathematics works"*, cross loaded on both factors and was subsequently removed from the analyses. Responses to item 15 were proposed to demonstrate teaching efficacy. A possible reason this item may have cross loaded is that addressing whether or not one finds it difficult to explain a concept certainly has to do with a teacher's ability beliefs, however this question may also be addressing the teacher's thoughts on whether or not students will understand their explanation, potentially leading participants to approach this question from both a teaching efficacy and outcome expectancy prospective. Additionally, these results may have to do with less emphasis being placed on using manipulatives in middle and high school level mathematics versus elementary school, thus potentially leading participants to have mixed responses in answering this question.

The newly developed knowledge of algebra for teaching framework (McCrory et al., 2012) was proposed to assess three factors related to teaching algebra; school knowledge, advanced knowledge, and teaching knowledge. Results from the KAT factor analysis were surprising in that those factors were not easily identifiable; thus, further analysis was conducted using KAT as one component. Similar studies examining the factor structure of the mathematics knowledge for teaching (MKT) framework (Hill, 2010) found that MKT items failed to load onto hypothesized factors - common content knowledge, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching, also leading researchers to combine all MKT items into one indicator. In the present study, when three factors were extracted, the first component displayed five teaching knowledge items and two school knowledge items. The second component contained four advanced knowledge items. The only advance knowledge item that did not load onto the second component was item 9, *"How many solutions exist for the equation* $tan(x)=x^2$*?"*. The third component included two school knowledge items, one teaching knowledge item, and one advanced knowledge item. Inferences can be made that the teaching knowledge and school knowledge items are problematic as they were found on similar factors. Additionally, these results support recent developments (Reckase et al., 2015) which acknowledge that there is no way to know if examinees will approach an item as writers intended, leading to modest support for distinct dimensions of the KAT instrument. Due to the issues with the factor structure and reliability of the school knowledge, advanced knowledge, and teaching knowledge components of the KAT instrument, interpretations should be made using KAT as a one component instrument.

Relations between MTEBI scale and KAT scale. To meet the first research goal of investigating whether there were differences in mathematics teachers KAT, mean scores and standard deviation on the KAT items were calculated. The descriptive analyses revealed that teachers had moderate overall KAT (*M*=.55, *SD*=.20). Although the present study did not utilize the three proposed factors of KAT in regression analyses or mediation analyses due to the factor structure of KAT, when taking into account the results of the Huang & Kulm, (2012) study which assessed the three KAT subscales, it was found that KAT scores in the present study produced similar mean scores. Scores observed in the present study, school knowledge (*M*= .68), advanced knowledge *(M=.37),* and teaching knowledge (*M*=.59) were comparable to those from perspective middle school mathematics teachers (Huang & Kulm, 2012), school knowledge (*M*=.61), advanced knowledge (*M*=.20), and teaching knowledge (*M*=.49). Most notably are the low advanced knowledge scores which suggests that secondary mathematics teachers do not demonstrate a deep understanding of school algebra concepts including calculus, linear algebra, number theory, abstract algebra, real and complex analysis, and mathematical modeling. Overall results indicate that knowledge of algebra for teaching remains an area of difficulty for secondary mathematics teachers.

Findings based on bivariate correlations provide support that portions of the MTEBI scale are related to the KAT scale. When examining the relation between participants MTEBI and KAT scores it was found that teaching efficacy scores were not correlated to participant's KAT scores, and outcome expectancy scores were negatively correlated to KAT scores. As discussed in previous chapters, the KAT construct is fairly new with fewer than five original studies in publication which include the instrument in its research. However, prior studies have investigated teacher's mathematics knowledge using the Learning Mathematics for Teaching (LMT) instrument which measures Mathematics Knowledge for Teaching (MKT) in relation to MTEBI scores. Specifically, Swars et al., (2007) found that there were no relationships between content knowledge scores and MTEBI scores, which contradict the findings in the present study. The present finding confirms that teacher's outcome expectancy beliefs are negatively related with their teaching knowledge, indicating that as participants increased their belief that they can influence students learning, their algebra knowledge declined. A possible explanation for this finding could be that a teacher who has high KAT may be aware of the difficulty that algebra poses for students, thus leading the teacher to have unstable outcome expectancy beliefs. Although there are no previous studies which specifically examine KAT in its relation to outcome expectancy, the results of the present study differ from Swars et al., (2007) which found no significant relationship between elementary mathematics teacher's outcome expectancy scores and specialized mathematics content knowledge scores. As stated previously, instruments and target populations were not identical to the present study, however it is important to note the differences in the findings. A contribution of the current study is that to date outcome expectancy beliefs have not been directly linked to knowledge of algebra for teaching.

Teacher characteristics. The present study explored various demographic characteristics to determine relationships among teachers results on MTEBI and KAT measures. Effect sizes are reported based on Cohen's d (Cohen, 1988) in which a small, medium, and large effect size is considered d = .20, .50, and .80 respectively. Relevant findings are discussed in the current section.

High School Teachers. Based on overall results, participants who identified as high school teachers had a much greater knowledge of algebra for teaching than their middle school counterparts. Mean scores for high school teachers were (M=.62, SD=.19) while mean scores for middle school teachers were (M=.44, SD=.16). Further, Cohen's effect size value (d=1.02) suggests a large effect. The strong relationship found between being a high school teacher and KAT scores should be noted. One possible explanation for this finding is that the knowledge required to obtain a high school level mathematics teaching certification goes far beyond algebra so it may be expected that high school teachers would have higher advanced level knowledge than their peers who are middle school teachers. This finding is consistent with results in a similar study from Hill and Lubienski (2007) which examined the relationship between the specialized mathematical knowledge of Kindergarten-8th grade teachers in which grade level taught significantly predicted content knowledge scores, indicating that the higher the grade level taught, the better a teacher's mathematical content knowledge. There was no indication, however, that being a high school teacher was the cause of increased teaching efficacy as was the case in the Klassen and Chiu (2010) study as participants produced similar teaching efficacy scores (M=4.40, SD=.35) for middle school teachers and (M=4.34, SD=.43) for high school teachers. The absence of teaching grade level playing a role in teaching efficacy is consistent with previous research from Guskey (1988) which found no difference in grade level assignment as a function of efficacy.

Math Teaching Certification. Among teachers taking the MTEBI, there was a statistically significant relationship between MTEBI scores and math teaching certification. Overall, self-report teaching efficacy measures indicated that secondary mathematics teachers who hold a math teaching certification were highly efficacious. In comparing the means, standard deviations, and effect sizes, differences between teachers without a math teaching certification (*M*=4.20, *SD*=.44) and teachers with math teaching certification (*M*=4.42, *SD*=.37), represented a medium effect size with a Cohen's d of .49. One possible explanation for this finding is a teacher who chooses to earn a math certification should be well equipped to teach the subject and therefore may display higher teaching efficacy as opposed to teachers who have not earned a math teaching certification. Additionally, it was found that teachers with a math teaching certification earned higher KAT scores (*M*=.57, *SD*=.19) than teachers without a math teaching certification (*M*=.45, *SD*=.21), representing a medium effect size with a Cohens d of .60. Which confirms the assertion of (Brown, Davis, and Kulm, 2011) that teachers who are certified to teach mathematics have stronger pedagogical and mathematics knowledge.

Years of Education. In the present study, teachers with more years of education displayed higher teaching efficacy belief scores. In comparing the means, standard deviations, and effect sizes, differences between the teaching efficacy of teachers who earned a high school diploma (*M*=3.77, *SD*=.55) and those who earned a bachelor's degree (*M*=4.33, *SD*=.37) represented a large effect size (d= 1.19); differences between teachers who earned a high school diploma (*M*=3.77, *SD*=.55) and those who earned a master's degree (*M*=4.43, *SD*=.34) also represented a large effect size (d= 1.44); differences between teachers who earned a high school diploma (*M*=3.77, *SD*=.55) and those who earned a doctorate degree (*M*=4.83, *SD*=.23), represented the largest effect size (d= 2.51). Observed differences between the teaching efficacy of teachers who earned a bachelor's degree (*M*=4.33, *SD*=.37) and those who earned a master's degree

(*M*=4.43, *SD*=.34), represented a small effect size (d= 0.28); and differences between teachers who earned a bachelor's degree (*M*=4.33, *SD*=.37) and those who earned a doctorate degree (*M*=4.83, *SD*=.23), represented a large effect size (d=1.62). Lastly, differences between the teaching efficacy of teachers who earned a master's degree (*M*=4.43, *SD*=.34) and those who earned a doctorate degree (*M*=4.83, *SD*=.23), displayed a large effect size (d=1.38).

It should be noted that 47% of participants indicated they had earned a bachelor's degree, 44% indicated they had earned a master's degree, and 4% of participants indicated they had earned a doctorate degree, leaving a small number of participants, 5%, who had not earned a postsecondary degree. There may be many reasons why years of education is positively associated with teaching efficacy beliefs. One possible explanation can be that individuals who have postsecondary degrees have learned more about teaching throughout their studies, thus they may exert a positive outlook on their teaching capabilities.

Years of Experience. Replicating findings from previous studies (Klassen and Chiu, 2010; Ross, Cousins, and Gallada, 1996) results showed that years of experience was related to teaching efficacy. It is important to note that teachers with greater years of experience exhibited higher outcome expectancy scores than teachers with lower years of experience. The findings, however, conflict with previous research (Desouza et al., 2004) which discovered that the more years a teacher taught, the lower their outcome expectancy scores were, (Hassan and Taraib, 2012) which demonstrated a low relationship between years of teaching experience and teacher's outcome expectancy beliefs, and (Ghaith and Yaghi, 1997) which found that teachers experience was negatively correlated with teaching efficacy. Previous studies from Hill (2007) found that amongst middle school mathematics teachers, those with more years of experience teaching had higher levels of teaching specific mathematical knowledge, which was not found to be true in the present study.

Public School Teachers. Initial findings confirm that KAT scores were found to have a significant but negative correlation amongst responses from those who identify as public school teachers, which were 86% of participants. Upon comparison of the means, standard deviations, and effect sizes, differences between teachers who identified as public school teachers (M=.53, SD=.20) and those who were not public school teachers (M=.63, SD=.18) suggested a medium effect based on Cohen's d effect size value (d=.53). Thus, indicating that being a public school teacher negatively contributed to KAT scores. A possible explanation for these results could stem from the environment in which a public school teacher and non-public school teacher work. Due to state and government requirements, all children are guaranteed a free and public education. This service is provided by public school districts. With that in mind, a public school teacher may potentially receive new students on a rolling basis throughout the school year making it difficult to establish and maintain among student groups. Contrarily, a non-public school may have an enrollment capacity or restrictions on receiving new students, leading a non-public school teacher to have a consistent group of students in their class for an entire school year. This discrepancy in the public school and non-public school environment may potentially lead to varying outcome expectancies (an expectation that teaching will influence student's learning) among these groups.

Teacher Type. The present study is unique in that it investigated three subgroups of teachers, in-service, pre-service, and former teachers. In comparing the means, standard deviations, and effect sizes, differences between teachers scores on the MTEBI and KAT showed that preservice teachers generally had close scores with the other subgroups. Pre-service teachers displayed teaching efficacy scores (M=4.04, SD=.58), outcome expectancy scores (M=3.67, SD=.48), and knowledge of algebra for teaching scores (M=0.59, SD=.23). In examining differences among teaching efficacy scores between pre-service and in-service teachers a medium effect (d=.72) was represented and when comparing teaching efficacy scores among pre-service and former teachers a medium effect (d=.73) was observed. Possible reasons for the lower teaching efficacy results than their experienced counterparts are that pre-service teacher's knowledge base is impacted heavily by their university experience which may or may not include field experience in the classroom. When examining differences in outcome expectancy among pre-service and in-service teachers a small effect (d = .37) was observed and a small effect (d = .23) was observed among pre-service and in-service teachers KAT scores.

What is interesting in these results is that in-service mathematics teachers displayed higher teaching efficacy but lower outcome expectancy and knowledge of algebra for teaching than pre-service teachers with teaching efficacy (M=4.39, SD=.35), outcome expectancy (M=3.49, SD=.49), and knowledge of algebra for teaching (M=0.53, SD=.20). Due to pre-service teachers lack of classroom based experiences, responses to the outcome expectancy questions are hypothetical whereas the responses of an in-service teacher are most likely linked to their actual teaching experiences. Pre-service teachers may exert optimism in regards to the potential effect they may have on a student's learning experiences thus leading to a high outcome expectancy. A pre-service teacher also may display higher algebra knowledge scores as they may be currently enrolled or recently enrolled in mathematics courses as part of their university coursework. When examining the differences among in-service and former teachers teaching efficacy a small effect (d=.12) was observed, and among outcome expectancy a medium effect (d=.50) was observed. Differences among in-service teachers and former teachers KAT showed a large effect (d=.81). Former mathematics teachers had the highest overall scores amongst all groups with teaching efficacy (M=4.44, SD=.49), outcome expectancy (M=3.78, SD=.65), and knowledge of algebra for teaching (M=0.68, SD=.17). When observing differences between pre-service teachers and former teachers' outcome expectancy a small effect (d=.19) was displayed. Differences in KAT scores among pre-service teachers and former teachers displayed a nearly medium effect (d=.49).

Outcome expectancy. In all teacher groups; pre-service teachers, in-service teachers, and former teachers, what became evident was the lack of relation between teaching efficacy and outcome expectancy scores. Teachers tended to have a high teaching efficacy but a lower outcome expectancy indicating that although teachers may have high teaching ability beliefs, they do not believe that they can influence student learning. Theoretically, it has been shown that mathematics teaching efficacy and outcome expectancy are two distinct constructs so these results are reasonable in the theoretical context. Although the lack of relation is in line with the theory, practically this is perplexing. If a teacher has high ability beliefs, why wouldn't these beliefs transfer to their ability to influence student outcomes through their teaching? Schunk (2012) contends that individuals who perform well have confidence in their capabilities or efficacy beliefs and expect positive outcomes for their efforts, although this was not shown to be the case in the present study. Because of this discrepancy, additional emphasis should be placed on the outcome expectancy construct as it relates to secondary mathematics teachers.

Mediation analyses inferences. Analyses testing the direct, indirect, and total effects of teaching characteristics on MTEBI through KAT resulted in identifying a mediational relationship between variables. Several of the direct paths from teacher characteristics (years of experience, years of education, and age) to teaching efficacy were significant. This suggests that individuals who differ by one unit on teacher characteristics but are equal on KAT are estimated to differ in the positive direction on teaching efficacy. Additionally, the direct path from teacher characteristics (public school teacher) to outcome expectancy was significant, which suggests that individuals who differ by one unit on teacher characteristics but are equal on KAT are estimated to differ in the negative direction on outcome expectancy.

The indirect path failed to identify knowledge of algebra for teaching as a mediator between teaching characteristics and teaching efficacy. Several mediation analyses were conducted by entering in single independent variables which included various teacher characteristics that were found to be correlated to teaching efficacy (math certification, years of education, teacher type, and age). After analyzing several teaching efficacy mediation models, it was determined that none of the models had a significant indirect effect with KAT indicating that knowledge of algebra for teaching scores did not mediate the relationship between teaching characteristics and teaching efficacy scores. However, knowledge of algebra for teaching was identified as a mediator between teaching characteristics and outcome expectancy. Ultimately, it was found that the indirect path from public school teachers to outcome expectancy was mediated by passing through KAT.

Limitations

The first limitation of the study was the length of the survey instrument. Included were a 27-item demographic questionnaire, a 21-item teaching efficacy/outcome expectancy questionnaire, and a 15-item algebra questionnaire. It is possible that testing fatigue may have been a factor in the participant's responses. Additionally, some participants who started the survey did not complete the survey.

The second limitation was the use of self-report data. With participants knowing the intent of the teaching efficacy/outcome expectancy instrument, they may have tried to answer the questions favorably. Additionally, the demographic survey included self-report items such as age, gender, ethnicity, etc. This could be problematic as there is no way to verify the accuracy of the participant's responses.

The third limitation was the sample selection. The researcher identified the target population, mathematics teachers, and distributed the survey via email to potential participant's work and school related email addresses. Since the survey was distributed online, other teachers had the opportunity to forward the survey to other qualified participants. This can be a limitation because participants were from various school districts in multiple states and countries which may make it difficult to generalize the findings.

Despite these limitations, the current study extends our understandings of the relationship of teaching efficacy, outcome expectancy, and knowledge of algebra for teaching. Furthermore, the current study integrated concepts from social cognitive theory (self-efficacy and outcome expectancy) and pedagogical content knowledge theories to further understand how these relationships interact with one another. This study also adds to the knowledge of algebra for teaching literature in identifying areas for future research as discussed in the following section.

Recommendations for Future Research

There is much to consider in working toward discovering a relationship between teaching efficacy, outcome expectancy, and knowledge of algebra for teaching among secondary mathematics teachers. Continued research is necessary in order to further identify the extent of this relationship.

Future research should examine teaching efficacy, outcome expectancy, and knowledge of algebra for teaching in relation to outcomes unrelated to student achievement such as teacher retention and teacher effectiveness. When examining teacher's content knowledge, in this case KAT, many prior studies explore content knowledge strictly in relation to student achievement. This too is true for studies that examine teacher's efficacy and outcome expectancy beliefs, they are typically gauged by student's success. This is not unreasonable given the core of any educational system is driven by student learning, however it has been studied many times over. Future research should use MTEBI and KAT scores to predict math teacher retention and enhance professional development for current teachers. Specifically, these results suggest that professional education efforts might focus on how to influence a teacher's outcome expectancy as a means to effect student outcomes. Then it may be acceptable to use the results as a way of determining if a teacher is truly equipped to teach secondary mathematics rather than relying predominately on passing a state licensure exam to make personnel decisions.

Another area for future research would be to conduct a longitudinal study to examine any changes that may occur over time in teaching efficacy, outcome expectancy, or knowledge of algebra for teaching. Possible collection points could be at the beginning, middle, and end of a school year.

Additionally, it is recommended that future researchers look further into the knowledge of algebra for teaching instrument to determine if valid inferences can be made based on the three proposed subscales– school knowledge, advanced knowledge, and teaching knowledge. Based on analyses conducted in the present study, individual KAT subscales were difficult to identify.

Finally, the most practical area for further research is to target a specific cluster of mathematics teachers such as within the same state, school district, or within a cohort of teacher leaders (such as mathematics department heads) in efforts to generalize the findings.

Implications

Considerable research has examined teaching efficacy and outcome expectancy beliefs at the beginning stages of a teacher's career (i.e. pre-service teachers). Developing an understanding of how an individual teachers teaching efficacy and outcome expectancy differs across the span their career is worth studying. Research on the knowledge required for secondary mathematics teachers, namely knowledge of algebra for teaching, to date has been understudied. Information gained by this study supports the idea that outcome expectancy is related to knowledge of algebra for teaching, but not teaching efficacy. The present study extends the research by highlighting how knowledge of algebra for teaching, teacher characteristics (years of experience, teaching grade level, and having a math teaching certification), were related to teaching efficacy and outcome expectancy. Further research is needed to determine the underlying factors causing an increase or decrease in teaching efficacy, outcome expectancy, and knowledge of algebra for teaching.

Conclusion

This study investigated the relationship between teacher's mathematics teaching efficacy beliefs, outcome expectancy beliefs, and knowledge of algebra for teaching of pre-service, in-service, and former mathematics teachers. To date, teaching efficacy has been shown to be related to student outcomes such as achievement (Ross, 1994), yet few studies have investigated teaching efficacy in terms of teacher outcomes. Thus, the intent of the study was to contribute to the knowledge base of teaching efficacy, outcome expectancy, and knowledge of algebra for teaching specifically among middle and high school mathematics teachers as they are an understudied population.

Significant relationships were found amongst mathematics teaching efficacy beliefs and having a math teaching certification and increased years of experience; outcome expectancy beliefs and being a public school teacher; knowledge of algebra for teaching and ethnicity, identifying as a public school teacher, and high school teacher; and knowledge of algebra for teaching scores and outcome expectancy scores. Additionally, knowledge of algebra for teaching acted as a mediator between outcome expectancy scores and identifying as a public school teacher.

The findings of the study provide many ideas for future research in the area of teaching efficacy, outcome expectancy, and specific content knowledge required for teaching secondary level mathematics. Further research regarding the knowledge of algebra for teaching construct is warranted. Knowledge gained through further studies will allow researchers to hone in on the characteristics necessary to prepare educators to teach secondary mathematics.

Appendix A

Consent Form

UNIVERSITY OF HOUSTON
CONSENT TO PARTICIPATE IN RESEARCH

TEACHING EFFICACY AND KNOWLEDGE FOR ALGEBRA TEACHING AMONG SECONDARY MATHEMATICS TEACHERS

You are being invited to participate in a research project conducted by *Ashley Warren* from the *Educational Psychology and Individual Differences* department at the University of Houston. *This project is part of a dissertation research under the supervision of Dr. Weihua Fan.*

NON-PARTICIPATION STATEMENT

Your participation is voluntary and you may refuse to participate or withdraw at any time without penalty or loss of benefits to which you are otherwise entitled. You may also refuse to answer any question. *If you are a student, a decision to participate or not or to withdraw your participation will have no effect on your standing.*

PURPOSE OF THE STUDY

The purpose of the project is to identify teacher efficacy beliefs, outcome expectancy beliefs, and knowledge of algebra for teaching.

PROCEDURES

You will be one of approximately 100 subjects to be asked to participate in this project. Data will be collected using quantitative measures. Anonymous self-report surveys will be distributed via paper and online measures to all participants.

- Number of interactions: One
- Requirements of the research subject: Survey questionnaires

- Time commitment: 40-60min

CONFIDENTIALITY

Your participation in this project is confidential and your responses will remain anonymous.

RISKS/DISCOMFORTS

There are no forseeable risks associated with this project.

BENEFITS

While you will not directly benefit from participation, your participation may help investigators better understand teacher efficacy in mathematics, which can provide implications for students academic outcomes, necessity for professional development amongst teachers, and levels of content knowledge in teachers.

INCENTIVES/REMUNERATION

Potential subjects will have the option of submitting their contact information into a drawing for a $25 gift card.

ALTERNATIVES

Participation in this project is voluntary and the only alternative to this project is non-participation.

PUBLICATION STATEMENT

The results of this study may be published in professional and/or scientific journals. It may also be used for educational purposes or for professional presentations. However, no individual subject will be identified.

If you have any questions, you may contact Ashley Warren at awarren2@uh.edu. You may also contact Dr. Weihua Fan faculty sponsor, at wfan2@uh.edu.

ANY QUESTIONS REGARDING YOUR RIGHTS AS A RESEARCH SUBJECT MAY BE ADDRESSED TO THE UNIVERSITY OF HOUSTON COMMITTEE FOR THE PROTECTION OF HUMAN SUBJECTS (713-743-9204).

Appendix B

Demographic Survey

Demographic Survey

Please complete this survey after signing the informed consent form.

1. Please indicate which one of the following describes you

__ Current middle or high school math teacher	__ Former middle or high school math teacher	__Pre-service middle or high school math teacher

If the above choices are not applicable, you do not need to continue the survey.

2. Please indicate your gender.

__Male __Female

3. Please indicate your age below.

____

4. Please specify your ethnicity.

__White	__Hispanic or Latino	__Black or African American
__Native American or American Indian	__Asian or Pacific Islander	__Other

5. What is the highest degree or level of school you have completed? If currently enrolled, highest degree received.

__High school diploma	__Associates	__Bachelors
__Masters	__Doctorate	__Other ________________

In-service secondary mathematics teachers please proceed to Section A and complete questions #6-14
Pre-service secondary mathematics teachers please proceed to Section B and complete questions #15-21
Former secondary mathematics teachers please proceed to Section C and complete questions #22-27

Section A – Pre-Service Secondary Mathematics Teachers

6. Are you currently enrolled in a teacher preparation program at a college or university?

__Yes __No

a. If yes, please indicate your classification

__ Freshman __ Junior __Masters
__Sophomore __ Senior __ Doctoral

b. Please indicate your cumulative GPA

__ 3.5 – 4.0 __ 2.0 – 2.9
__3.0 – 3.5 __ Less than 2.0

7. Please indicate the grade(s) to which you are presently preparing to teach mathematics. (Check ALL that apply):

__ Grades K-2 __ Grade 6 __ Grade 10

__ Grade 3 __ Grade 7 __ Grade 11

__ Grade 4 __ Grade 8 __ Grade 12

__ Grade 5 __ Grade 9 ___ I do not presently teach math/I am not presently preparing to teach math

a. If you currently teach in a content specific area, please indicate the content below:

__Algebra I __ Pre-Calculus __ Not Applicable
__ Algebra II __ Calculus
__ Geometry __ Math Models w/Applications

8. Please indicate the grade(s) in which you <u>have ever taught</u> mathematics. (Check ALL that apply):

__ Grades K-2 __ Grade 6 __ Grade 10

__ Grade 3 __ Grade 7 __ Grade 11

__ Grade 4 __ Grade 8 __ Grade 12

__ Grade 5 __ Grade 9 ___ I have never taught mathematics

a. If you have ever taught in a content specific area, please indicate the content below:

__Algebra I __ Pre-Calculus __ Not Applicable
__ Algebra II __ Calculus
__ Geometry __ Mathematical Models with Applications

9. Please indicate the type of school in which you are currently completing field experience *(check all that apply)*

__Public __Private

__ Charter __ N/A

10. Do you have a math coach or math specialist at your school?

__Yes __No

If yes, please indicate how often you receive(d) training, professional development, and/or assistance related to content knowledge and/or teaching:

__Daily __Weekly __Monthly

__Quarterly __Yearly __ N/A

11. For what grade levels will you receive your teaching certification? (Check ALL that apply.)

__ Grades K-2
__ Grade 3
__ Grade 4
__ Grade 5
__ Grade 6
__Grade 7
__ Grade 8
__ Grade 9
__ Grade 10
__ Grade 11
__ Grade 12
__ N/A

12. For what subject matters will you receive your teaching certification? (Check ALL that apply.)

__ All subjects (generalist)
__ Mathematics
__ Science
__ Other subject matter-specific certification: ______________________________
__ N/A

13. Please indicate your certification status: (Check ONE answer)

__ Emergency

__ Temporary

__ One-Year

__ Probationary/Preliminary

__ Standard (full/completed)

__Not certified

14. Please indicate the number of years you have taught middle or high school mathematics:

__no experience	__0.5 yrs.	__1 yr.	__ 1.5 yrs.	__2 yrs.	__2.5 yrs.	__3 yrs.	__3.5 yrs.	__4 yrs.
__4.5 yrs.	__5 yrs.	__5.5 yrs.	__6 yrs.	__ 6.5 yrs.	__7 yrs.	__7.5 yrs.	__8 yrs.	__ 8.5 yrs.
__9 yrs.	__9.5 yrs.	__ 10 yrs.	__10.5 yrs.	__11 yrs.	__11.5 yrs.	__ 12 yrs.	__ 12.5 yrs.	__ 13 yrs.
__ 13.5 yrs.	__ 14 yrs.	__ 14.5 yrs.	__15 yrs.	__More than 15 yrs.				

Section B - Current Secondary Mathematics Teachers

15. Please indicate the grade(s) to which you presently teach mathematics. (Check ALL that apply):

__ Grades K-2	__ Grade 6	__ Grade 10

__ Grade 3 __ Grade 7 __ Grade 11

__ Grade 4 __ Grade 8 __ Grade 12

__ Grade 5 __ Grade 9 ___ I do not presently teach math

a. If you currently teach in a content specific area, please indicate the content below:

__Algebra I __ Pre-Calculus __ Not Applicable

__ Algebra II __ Calculus

__ Geometry __ Math Models w/Applications

16. Please indicate the grade(s) in which you have ever taught mathematics. (Check ALL that apply):

__ Grades K-2 __ Grade 6 __ Grade 10

__ Grade 3 __ Grade 7 __ Grade 11

__ Grade 4 __ Grade 8 __ Grade 12

__ Grade 5 __ Grade 9 ___ I have never taught mathematics

a. If you have ever taught in a content specific area, please indicate the content below:

__Algebra I __ Pre-Calculus __ Not Applicable

__ Algebra II __ Calculus

__ Geometry __ Mathematical

Models with
Applications

17. Do you have a math coach or math specialist at your school?

__Yes __No

If yes, please indicate how often you receive(d) training, professional development, and/or assistance related to content knowledge and/or teaching:

__Daily __Weekly __Monthly

__Quarterly __Yearly __ N/A

18. For what grade levels did you receive your teaching certification? (Check ALL that apply.)

__ Grades K-2
__ Grade 3
__ Grade 4
__ Grade 5
__ Grade 6
__Grade 7
__ Grade 8
__ Grade 9
__ Grade 10
__ Grade 11
__ Grade 12
__ N/A

19. For what subject matters did you receive your teaching certification? (Check ALL that apply.)

__ All subjects (generalist)
__ Mathematics
__ Science
__ Other subject matter-specific certification: ______________________________
__ N/A

20. Please indicate your certification status: (Check ONE answer)

__ Emergency

__ Temporary

__ One-Year

__ Probationary/Preliminary

__ Standard (full/completed)

__Not certified

21. Please indicate the number of years you have taught middle or high school mathematics:

__no experience	__0.5 yrs.	__1 yr.	__ 1.5 yrs.	__2 yrs.	__2.5 yrs.	__3 yrs.	__3.5 yrs.	__4 yrs.
__4.5 yrs.	__5 yrs.	__5.5 yrs.	__6 yrs.	__ 6.5 yrs.	__7 yrs.	__7.5 yrs.	__8 yrs.	__ 8.5 yrs.
__9 yrs.	__9.5 yrs.	__ 10 yrs.	__10.5 yrs.	__11 yrs.	__11.5 yrs.	__ 12 yrs.	__ 12.5 yrs.	__ 13 yrs.
__ 13.5 yrs.	__ 14 yrs.	__ 14.5 yrs.	__15 yrs.	__More than 15 yrs.				

Section C – Former Secondary Mathematics Teachers

22. Please indicate the grade(s) to which you formerly taught mathematics. (Check ALL that apply):

__ Grades K-2	__ Grade 6	__ Grade 10
__ Grade 3	__ Grade 7	__ Grade 11

__ Grade 4 __ Grade 8 __ Grade 12

__ Grade 5 __ Grade 9

a. If you taught in a content specific area, please indicate the content below:

__Algebra I __ Pre-Calculus __ Not Applicable

__ Algebra II __ Calculus

__ Geometry __ Math Models w/Applications

23. Did you have access to a math coach or math specialist at your former school of employment?

__Yes __No

If yes, please indicate how often you receive(d) training, professional development, and/or assistance related to content knowledge and/or teaching:

__Daily __Weekly __Monthly

__Quarterly __Yearly __ N/A

24. For what grade levels did you receive your teaching certification? (Check ALL that apply.)

__ Grades K-2 __ Grade 8

__ Grade 3 __ Grade 9

__ Grade 4 __ Grade 10

__ Grade 5 __ Grade 11

__ Grade 6 __ Grade 12

__Grade 7 __ N/A

25. For what subject matters did you receive your teaching certification? (Check ALL that apply.)

__ All subjects (generalist)

__ Mathematics

__ Science

__ Other subject matter-specific certification: ____________________________

__ N/A

26. Please indicate your certification status: (Check ONE answer)

__ Emergency

__ Temporary

__ One-Year

__ Probationary/Preliminary

__ Standard (full/completed)

__Not certified

27. Please indicate the number of years you have taught middle or high school mathematics:

__no experience	__0.5 yrs.	__1 yr.	__ 1.5 yrs.	__2 yrs.	__2.5 yrs.	__3 yrs.	__3.5 yrs.	__4 yrs.
__4.5 yrs.	__5 yrs.	__5.5 yrs.	__6 yrs.	__ 6.5 yrs.	__7 yrs.	__7.5 yrs.	__8 yrs.	__ 8.5 yrs.
__9 yrs.	__9.5 yrs.	__ 10 yrs.	__10.5 yrs.	__11 yrs.	__11.5 yrs.	__ 12 yrs.	__ 12.5 yrs.	__ 13 yrs.
__ 13.5 yrs.	__ 14 yrs.	__ 14.5 yrs.	__15 yrs.	__More than 15 yrs.				

Appendix C

Mathematics Teaching Efficacy Beliefs Instrument

Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)

Enochs, Smith, & Huinker (1999)

Instructions: Please circle the response that best indicates how you feel about each statement below.

1=Strongly Disagree 2=Disagree 3=Uncertain 4=Agree 5=Strongly Agree

Item#	Statement					
1.	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort	1	2	3	4	5
2.	I am continually finding better ways to teach mathematics	1	2	3	4	5
3.	Even if I try very hard, I will not teach mathematics as well as I will most subjects	1	2	3	4	5
4.	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach	1	2	3	4	5
5.	I know how to teach mathematics concepts effectively	1	2	3	4	5
6.	I am not very effective in monitoring mathematics activities	1	2	3	4	5
7.	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching	1	2	3	4	5
8.	I generally teach mathematics ineffectively	1	2	3	4	5
9.	The inadequacy of a student's mathematics background can be overcome by good teaching	1	2	3	4	5
10.	When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher	1	2	3	4	5
11.	I understand mathematics concepts well enough to be effective in teaching mathematics	1	2	3	4	5
12.	The teacher is generally responsible for the achievement of students in mathematics	1	2	3	4	5
13.	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching	1	2	3	4	5

14.	If parents comment that their child is showing more interest in mathematics in school, it is probably due to the performance of the child's teacher	1	2	3	4	5
15.	I find it difficult to use manipulatives to explain to students why mathematics works	1	2	3	4	5
16.	I typically am able to answer students' questions about mathematics	1	2	3	4	5
17.	I wonder if I have the necessary skills to teach mathematics	1	2	3	4	5
18.	Given a choice, I would not invite the principle to evaluate my mathematics teaching	1	2	3	4	5
19.	When a student has difficulty understanding a mathematics concept, I usually am at a loss as to how to help the student understand better	1	2	3	4	5
20.	When teaching mathematics, I usually welcome student questions	1	2	3	4	5
21.	I do not know what to do to turn students on to mathematics	1	2	3	4	5

Appendix D

Knowledge of Algebra for Teaching

Survey of Knowledge for Teaching Algebra

Michigan State University
2006

Assessment Questions

Form 1

* Please mark your choices and write your responses in the Answer Booklet
* Please return this form to the administrator together with your Answer Booklet after completion

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

1. At a storewide sale, shirts cost \$8 each and pants cost \$12 each. If S is the number of shirts and P is the number of pants bought, which of the following is a meaning for the expression 8S + 12P?

   A. The number of shirts and pants bought

   B. The cost of 8 shirts and 12 pants

   C. The cost of P shirts and S pants

   D. The cost of S shirts and P pants

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

2. Susan was trying to solve the equation $2x^2 = 6x$.

First she divided both sides by 2.

$$x^2 = 3x$$

Then she divided both sides by x:

$$x = 3$$

Gustavo said, “You can’t divide both sides by x.” Susan responded, “If you can divide both sides by 2, why can’t you divide by x?” They asked their teacher to explain.

Which of the following explanations is correct?

A. Since x is a variable it can vary, you may not be dividing both sides by the same number.

B. You can’t cancel x because it does not represent a real number.

C. You can only divide by whole numbers when solving equations.

D. It is better to take the square root of both sides after dividing by 2, that way you won’t have to worry about dividing by x.

E. If you divide both sides by x, then you might be dividing by 0, and would miss the solution $x = 0$.

Assessment Questions **Form 1** **Please mark your choice in the Ans wer Booklet**

3. Given the function f, defined by $f(x) = 3x^2 + 2x - 4$, which of the following equals $f(x + a)$?

   A. $3x^2 + 3a^2 + 2x + 2a - 4$

   B. $3x^2 + 6xa + 3a^2 + 2x - 4$

   C. $3(x + a)^2 + 2(x + a) - 4$

   D. $3(x + a)^2 + 2x - 4$

   E. $3x^2 + 2x - 4 + a$

Adopted from Arizona State University, Carlson et al.

Assessment Questions **Form 1** **Please mark your choice in the Ans wer Booklet**

4. Let $f(x) = \log_2 x^2$. Which of the following functions have the same graph as $y = f(x)$?

i. $y = 2\log_2 x$
ii. $y = 2\log_2 |x|$
iii. $y = 2|\log_2 x|$

A. i only

B. ii only

C. iii only

D. i and ii only

E. i, ii, and iii

Assessment Questions **Form 1** **Please mark your choice in the Ans wer Booklet**

5. Students are given the following problem:

Find the number of the real roots of the equation $9^x \square 3^x \square 6 = 0$

Peter denotes $y = 3^x$ and gets the equation $y^2 \square y \square 6 = 0$, which has 2 different roots. He concludes that the given equation also has 2 different roots.

Which of the following is true about Peter's solution?

A. Peter's conclusion and his arguments are correct.

B. Peter's original approach to the problem (substitution of $y = 3^x$) is not correct.

C. Peter factors wrong.

D. The quadratic equation $y^2 \square y \square 6 = 0$ does not have 2 different roots.

E. Peter does not take into account the range of the function $y = 3^x$.

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

6. Which of the following can be represented by areas of rectangles?

i. The equivalence of fractions and percents, e.g. $\frac{3}{5} = 60\%$
ii. The distributive property of multiplication over addition: For all real numbers a, b, and c, we have $a(b + c) = ab + ac$
iii. The expansion of the square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

A. ii only

B. i and ii only

C. i and iii only

D. ii and iii only

E. i, ii, and iii

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

7. A student is asked to give an example of a graph of a function $y = f(x)$ that passes through the points A and B (see Figure 1). The student gives the answer shown in Figure 2. When asked if there is another answer the student says: "No, this is the only function."

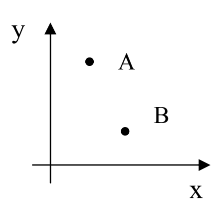


Figure 1

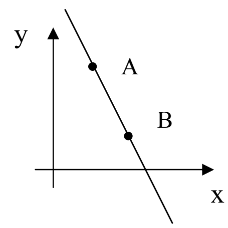


Figure 2

Which of the following best evaluates the student's answer of "No" to the second question?

A. The student is right, because that is the only way a line will pass through both points.

B. The student is right, because this function is of the form $f(x) = mx + b$.

C. The student is right, because his graph passes the vertical line test.

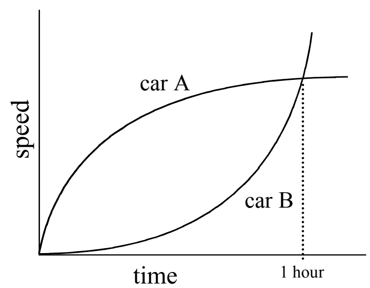
D. The student is wrong, because graphing is not an appropriate way to solve this problem.

E. The student is wrong, because there are infinitely many functions that pass through points A and B.

Adopted from Ruhama Evens paper.

Assessment Questions **Form 1** **Please mark your choice in the Ans wer Booklet**

8. The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are traveling in the same direction.) Use this information and the graph below to answer.



What is the relationship between the ***position*** of car A and car B at $t = 1$ hour?

A. The cars are at the same position.

B. Car A is ahead of car B.

C. Car B is passing car A.

D. Car A and car B are colliding.

E. The cars are at the same position and car B is passing car A.

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

9. How many solutions exist for the equation $\tan(x) = x^2$?

   A. No solutions

   B. Exactly one solution

   C. Exactly two solutions

   D. More than two solutions, but a finite number of solutions

   E. An infinite number of solutions

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

10. A textbook includes the following theorem:

> *If line l_1 has slope m_1 and line l_2 has slope m_2 then l_1 □ l_2 if and only if*
>
> *$m_1 \cdot m_2 = -1$ (i.e. " slopes are negative reciprocals").*
>
> *(McDougal Littell, Algebra 2)*

Three teachers were discussing whether or not this statement generalizes to all lines in the Cartesian plane.

Mrs. Allen: The statement of the theorem is incomplete: it doesn't provide for the pair of lines where one is horizontal and one is vertical. Such lines are perpendicular.

Mr. Brown: The statement is fine: a horizontal line has slope 0 and a vertical line has slope □ and it's OK to think of 0 times □ as –1.

Ms. Corelli: The statement is fine; horizontal and vertical lines are not perpendicular.

Whose comments are correct?

A. Mrs. Allen only

B. Mr. Brown only

C. Ms. Corelli only

D. Mr. Brown and Ms. Corelli.

E. None are correct.

Assessment Questions **Form 1** **Please mark your choice in the Ans wer Booklet**

11. In a first year algebra class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?

A. Talk about the rate of change of a graph of a line on an interval.

B. Talk about speed as distance divided by time.

C. Toss a ball in the air and use a motion detector to graph its trajectory.

D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.

E. Discuss the meaning of *m* in the graphs of several equations of the form $y = mx + b$.

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

12. Consider the statement below.

For all a and b in S, if ab = 0, then either a = 0 or b = 0.

For which of the following sets S is the above statement true?

i. the set of real numbers
ii. the set of complex numbers
iii. the set of 2x2 matrices with real number entries

A. i only

B. ii only

C. iii only

D. i and ii only

E. i, ii and iii

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

13. Some high school students were asked to prove that the following statement is true:

When you multiply any 3 consecutive whole numbers, your answer is always a multiple of 6.

Below are three answers.

Kate's answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3 as every third number is in the three times table.
Also, at least one number will be even and all even numbers are multiples of 2.
If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

Leon's answer

$1 \cdot 2 \cdot 3 = 6$

$2 \cdot 3 \cdot 4 = 24 = 6 \cdot 4$

$4 \cdot 5 \cdot 6 = 120 = 6 \cdot 20$

$6 \cdot 7 \cdot 8 = 336 = 6 \cdot 56$

Maria's answer

n is any whole number

$n \cdot (n+1) \cdot (n+2) = (n^2 + n) \cdot (n+2)$

$= n^3 + n^2 + 2n^2 + 2n$

Canceling the n's gives $1 + 1 + 2 + 2 = 6$

Which are valid proofs?

A. Kate's only

B. Maria's only

C. Kate's and Leon's

D. Leon's and Maria's

E. Kate's and Maria's

Adapted from Core Plus Mathematics Project.

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

14. How many real solutions does the following equation have?

$$\sqrt{x \square 2} = \sqrt{1 \square x}$$

- **A.** none
- **B.** one
- **C.** two
- **D.** four
- **E.** infinitely many

Assessment Questions **Form 1** **Please mark your choice in the Answer Booklet**

15. Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x+ y + z)^2$?

    i. See what happens in an example, such as $(3 + 4 + 5)^2$.
    ii. Use $(x + y + z)^2 = ((x+ y) + z)^2$ and the expansion of $(a + b)^2$.
    iii. Use the geometric model shown below.

	x	y	z
x	x^2	xy	xz
y	xy	y^2	yz
z	xz	yz	z^2

A. ii only

B. iii only

C. i and ii only

D. ii and iii only

E. i, ii and iii

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